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Answer Key

Correct Answer Shown

Align Bottom and Left Edge

1. What uses a group of methods to solve problems about the relations between things?
   - algebra
   - geometry
   - hieroglyphics

2. Which of these is the way to express multiplication?
   - (4)(5)
   - both
   - 4X

3. How could you express X multiplied by Y + 5?
   - XY + 5
   - Y + 5X
   - X(Y + 5)

4. What is another way of expressing a + b + c?
   - b + c + a
   - (a)(b)(c)
   - a(b + c)

5. How else could you express a(b+c)?
   - (a)(b)(c)
   - b + ac
   - ab + c
   - ab + ac

6. What is another way of expressing (a · b) · c?
   - both
   - a· (b· c)
   - (b· a)· c

7. What are three commonly used laws of algebra?
   - commutative associative distributive
   - addition subtraction multiplication
   - relativity quantum mechanics

1. What is the variable in $X + 3 = 7$?
   - ◯ X
   - ◯ 4
   - ◯ 3
   - ◯ 7

2. Which of these is a term?

2X+3Y=47	X+8=16	5X
◯	◯	◯

3. Which is the coefficient in 4AXY?
   - ◯ A
   - ◯ X
   - ◯ 4
   - ◯ Y

4. Solve $15X = 35$ for X.

$^{7}/_{3}$	2	525
◯	◯	◯

5. What is the solution to $2X + 4 = 12$?

$2X = 8$	$X = 8$	$X = 4$
◯	◯	◯

6. Solve $4X + 5 = 21$ for X.

$X = {}^{1}/_{4}$	$X = 4$	$X = 16$
◯	◯	◯

7. Which is the equation for 3 packages of an unknown weight and one 4-pound package which equals 19 pounds?

$4X(3) = 19$	$3X + 4 = 19$
◯	◯

8. What is the value of C if $F = 68^{o}$ in $C = {}^{5}/_{9}(F - 32)$?

36^{o}	20^{o}	180^{o}
◯	◯	◯

1. Solve $2X = 6$ for X.

3	$^1/_3$	12
⬭	⬭	⬭

2. Solve $\frac{1}{3}X = 9$ for X.

$^1/_3$	27	3
⬭	⬭	⬭

3. Solve $\frac{X}{4} = 2$ for X.

$^1/_2$	2	8
⬭	⬭	⬭

4. Solve $\frac{2}{X} = 4$ for X.

$^1/_2$	2	8
⬭	⬭	⬭

5. Solve $2 + 3 = 5X$ for X.

$^5/_6$	$^1/_5$	1
⬭	⬭	⬭

6. Solve $5X - 3 = 2$ for X.

-1	1	$^1/_5$
⬭	⬭	⬭

7. Solve $2(X - 1) = 4$ for X.

3	1	12
⬭	⬭	⬭

8. Solve $5(X - 3) = \frac{58 + 2}{2}$ for X.

3	9	45
⬭	⬭	⬭

1. In algebra a number with a minus sign in front of it is considered to be ________.
   - ◯ negative
   - ◯ positive
   - ◯ subtracted

2. What is the result of -3 - (+2)?
   - ◯ -1
   - ◯ +5
   - ◯ +6
   - ◯ -5

3. What is the result of 5 - (-4)?
   - ◯ +1
   - ◯ +9
   - ◯ -1
   - ◯ -9

4. What is the <u>product</u> of -7 and -3?
   - +21 ◯
   - -21 ◯
   - -11 ◯

5. What is the quotient of -24 divided by 6?
   - ◯ $1/4$
   - ◯ -4
   - ◯ $-1/4$
   - ◯ +4

6. What is the product of $(2X)(4X^2)$?
   - ◯ $8X^3$
   - ◯ $8X^2$
   - ◯ $6X^3$
   - ◯ $6X^2$

7. What is the result of $6X^2$ divided by $2X$?
   - ◯ $12X^2$
   - ◯ $3X$
   - ◯ $3X^2$
   - ◯ $12X^3$

8. What is the <u>collected product</u> of X times 8 + 3Y - 6?
   - 2X + 3XY ◯
   - 8X + 3XY - 6X ◯

D625 Ma5

1. In mathematics what is any collection of objects?

set | group | pieces

2. Which of these is the set of odd integers?

$\{1, 3, 5, 7, 9\}$ | $(1, 3, 5, 7, 9)$ | $1, 3, 5, 7, 9$

3. What is the solution set for $X^2 - 10X + 21 = 0$?

$\{X = 4\}$ | $\{X = 10\}$ | $\left\{\begin{matrix} X = 7 \\ X = 3 \end{matrix}\right\}$

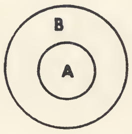
4. Which of these is the set of all values of X such that X is greater than 8?

$\{X > 8\}$ | $\{X \mid X > 8\}$ | $\{8 < X\}$

5. If $X = \{a, b, c, d\}$ and $Y = \{a, b, c, d\}$ which statement is true?

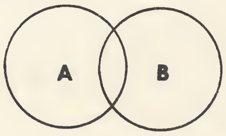
$X = Y$ | $X \neq Y$ | $X \sim Y$

6. In this diagram how would you describe A in relation to B?



a subset | a universal set | a set

7. In this diagram how would you describe A in relation to B?



a disjoint set | a subset | an overlapping set

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1. Solve $2X + 6 = 20$ for X.

   ◯ $X = 7$ ◯ $X = 14$ ◯ $X = 1/2$

2. Mary had $46 in her bank account, but after a withdrawal had a balance of only $24. Which equation would find the amount of withdrawal?

   ◯ $46+24=X$ ◯ $46+X=24$ ◯ $46-X=24$

3. What is the sum of -4, 2X, +8, 4Y, -3X?

   ◯ $-X+4Y+4$ ◯ $-5X+4Y+12$ ◯ $X+4Y-12$

4. What is the product of $(3X^3)(2X^4)$?

   ◯ $5X^7$ ◯ $6X^{12}$
   ◯ $5X^{12}$ ◯ $6X^7$

5. What is the quotient of $\frac{Y^{10}}{Y^2}$?

   ◯ Y^8 ◯ Y^{12}
   ◯ Y^5 ◯ Y^{20}

6. Multiply $-4X$ times $(3X+2Y-4)$.

   ◯ $12X+8XY-16X$ ◯ $-12X^2-8XY+16X$

7. What is the product of $(X+5)(X-3)$?

   ◯ $X^2 + 2X - 15$
   ◯ $2X + 5X + 2$
   ◯ $2X^2 + 8X + 15$

1. In algebra we deal with _____ and _____ quantities.
   - ◯ relative, non-relative
   - ◯ known, unknown
   - ◯ X, Y

2. Sara is 2 $^1/_2$ times as old as Jane. Jane is 10. What is the equation for Sara's age?

   ◯ $(2\frac{1}{2})(10)=S$ ◯ $10J=2\frac{1}{2}$ ◯ $2\frac{1}{2}S=10$

3. How old is Sara?

   ◯ 15 ◯ 20 ◯ 25

4. What is the exact value of X in X = 4Y?

   ◯ $\frac{4Y}{X}$ ◯ $\frac{X}{4Y}$ ◯ 4Y

5. Bill is paying twice as much as Joe per month and Mike is paying three times as much as Joe on their car. The total payment is $120. How would you find out how much each is paying?
   - ◯ X + Y + Z = 120
   - ◯ X + 2X + 3X = 120
   - ◯ X + 2Y + 3Z = 120

6. How much is each paying?
   - ◯ Joe=$20; Bill=$40; Mike=$80
   - ◯ Joe=$10; Bill=$60; Mike=$50
   - ◯ Joe=$20; Bill=$40; Mike=$60

1. Which of these is an "algebraic fraction"?

$\frac{2}{3}$ ◯ $\frac{1}{2X}$ ◯ $\frac{X}{2}$ ◯

2. What fraction is the same as $\frac{A+4}{2}$?

$\frac{3A + 12}{6}$ ◯ $\frac{3A + 12}{2}$ ◯ $\frac{3A + 4}{2}$ ◯

3. Reduce $\frac{(A + B)^2}{A^2 - B^2}$ to its lowest terms.

$\frac{A + B}{A - B}$ ◯ $\frac{A^2+2AB+B^2}{(A+B)(A-B)}$ ◯ $\frac{AB}{A - B}$ ◯

4. What is the result of $\frac{A}{4}$ divided by $\frac{A}{2B}$?

$\frac{A}{8B}$ ◯ $\frac{2AB}{4}$ ◯ $\frac{B}{2}$ ◯

5. What is the sum of $\frac{4}{3X} + \frac{1}{2X}$?

$\frac{5}{5X}$ ◯ $\frac{1}{X}$ ◯ $\frac{8X}{3X}$ ◯ $\frac{11}{6X}$ ◯

6. Combine $\frac{6}{A - B} + \frac{2}{B - A}$.

$\frac{8}{A - B}$ ◯ $\frac{8}{B - A}$ ◯ $\frac{4}{A - B}$ ◯

1. Solve $\frac{X}{4} + \frac{X}{3} = 21$ for X.

   - $X = 84$
   - $X = 1\ ^{3}/_{4}$
   - $X = 252$
   - $X = 36$

2. Joan bought a rug 10' by 12'. What was the total area of the rug? A=LW

   - 92
   - $1\ ^{1}/_{5}$
   - $^{5}/_{6}$
   - 120

3. Mr. Smith put $1000 in a savings account which accrued interest at 6% annually. He was paid interest semi-annually. How much interest did he receive in one year? i = prt

   - $60
   - $90
   - $30
   - $120

4. If Joe drove 350 miles at 70 mph, how long did the trip take? rate = miles/mph

   - 5 hours
   - 20 hours
   - 7 hours
   - 35 hours

5. Two trains 700 miles apart, left at the same time. The first train traveled 20 mph faster than the second. They met in 5 hours. Which equation gives the rate of the first train?

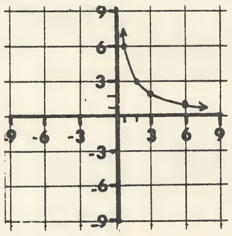
   - $R - 20 = \frac{700}{5}$
   - $R = \frac{700}{5}(R - 20)$
   - $R + (R - 20) = \frac{700}{5}$

1. What is the ratio of 6 inches to 2 feet?
   - ◯ $^1/_4$
   - ◯ $^6/_{24}$
   - ◯ $^6/_2$
   - ◯ $^3/_1$

2. What is a proportion?
   - ◯ a comparison by division
   - ◯ two ratios that are equal
   - ◯ a fraction

3. What is the product of the means in 6:18 = 1:3?
   - ◯ 6
   - ◯ 54
   - ◯ 18

4. Which equation does this graph satisfy?



   - ◯ $XY = 6$
   - ◯ $X^2 + 12X - 10 = 0$
   - ◯ $\frac{X}{Y} = 6$

5. Solve $\frac{2}{12} = \frac{3}{X}$ for X.
   - ◯ X = 18
   - ◯ X = 36
   - ◯ X = 2

6. What are the extremes in A:B = C:D?
   - ◯ B and C
   - ◯ A and C
   - ◯ A and D

1. Joan and Mary had 6 children between them. Joan had twice as many children as Mary. How many children did Joan have? J = 2M

   ◯ J = 2 ◯ J = 4
   ◯ J = 3 ◯ J = 5

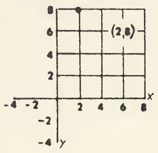
2. What's the value of Y in X = 4Y, if X = 0?

   ◯ 1 ◯ 2
   ◯ 0 ◯ 4

3. What's the value of of Y in X = 4Y, if X = 8?

   ◯ 2 ◯ 0
   ◯ 4 ◯ 1

4. Which is the graph of problem 3?



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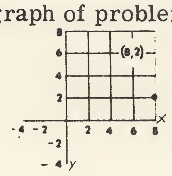

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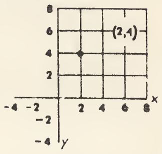

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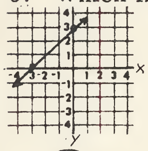
5. Which is the graph of X = Y + 3?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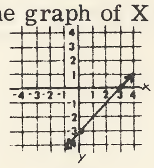
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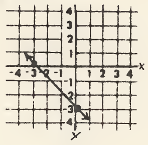
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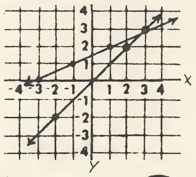
6. What's the solution to X = Y and X = 2Y - 3?



   ◯ (0, 0) ◯ (2, 1)
   ◯ (3, 3) ◯ (-1, 1)

1. You have a total of \$5000 to invest. Let L be your investment in Living Mutual Funds and F in a savings account in the Farmers' State Bank. Your dividend from Living is .07L and you get .05F from Farmers'. You received a dividend check from Living which was \$50 greater than your interest from Farmers'. What relationships do we know about L and F?

$L + F = 5000$	$.07L - L = .05F + F$
$.07L - .05F = 50$	$.07L + .05F = 5000$

2. What would be the first two steps in solving these simultaneous equations?
   - multiply bottom equation by 20, add result to top equation
   - find F in top equation, add result to bottom equation

3. What's the result of these two steps?
   $1.4L = 5000$ $1.4L + L = 6000$ $1.4L = 6000$

4. Solve for L and F.

$L = 6000$	$L = 3000$	$L = 2500$
$F = -1000$	$F = 2000$	$F = 2500$

5. Which symbols indicate 1)"greater than" and 2)"less than"?

1) $<$	1) $>$	1) $>$
2) $>$	2) $=$	2) $<$

1. What is the product of X^3 times X^4?

X ◯ X^{-1} ◯ X^{12} ◯ X^7 ◯

2. What is another way of writing $(4X^2Y^4)^3$?

$16X^5Y^7$ ◯ $4X^5Y^7$ ◯ $4X^6Y^{12}$ ◯ $64X^6Y^{12}$ ◯

3. What is the cube root of $27X^3$?

$3X^2$ ◯ $9X^2$ ◯ $3X$ ◯ $9X$ ◯

4. What is $\sqrt{54X^6 - 5X^6}$?

$\pm 7X^9$ ◯ $\pm 7X^3$ ◯ $\pm\sqrt{54X^3 - 5X^3}$ ◯

5. What is $(2X + 5)^2$?

$4X^2+25$ ◯ $4X^2+10X+25$ ◯ $4X^2+20X+25$ ◯

6. What is 92.478 to the nearest tenth?

◯ 92.4 ◯ 92.5
◯ 93 ◯ 94

7. What is the square root of the quantity "6 squared plus 8 squared"?

6 ◯ 8 ◯ 12 ◯ 10 ◯

1. Which is the standard form of $6X^2 + 3X = 15$?
   - ◯ $6X^2 = -3X + 15$
   - ◯ $6X^2 + 3X - 15 = 0$
   - ◯ $3X = -6X^2 + 15$

2. What is the general expression for the standard form of a quadratic equation?
   - ◯ $aX^2 + bX + c = 0$
   - ◯ $X^2 + X + 1 = 0$
   - ◯ $X^2 + 2X + 3 = 0$

3. $(X + 2)(X - 2)$ are the factors of $X^2 - 4 = 0$. What is the value of X?
   - ◯ 2
   - ◯ 4
   - ◯ ± 2
   - ◯ ± 4

4. Solve $X^2 - 10X + 21 = 0$ by factoring.

$X = -7$	$X = -5$	$X = 7$
$X = -3$	$X = -2$	$X = 3$
◯	◯	◯

5. What is the quadratic formula?
   - ◯ $\dfrac{b \pm \sqrt{b + 2a}}{4ac}$
   - ◯ $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   - ◯ $aX^2 + bX + c = 0$

6. Using the quadratic formula, find the solution for $3X^2 - 8X + 5 = 0$.

$X = 1$	$X = 3$	$X = 1$
$X = {}^5/_3$	$X = 5$	$X = 2$
◯	◯	◯

1. What is one root of $X^2 - 9X + 0 = 0$?

   ◯ 3 ◯ 1

   ◯ 2 ◯ 0

2. Solve $4X^2 - 10X + 6 = 0$ by factoring.

$X = 2$	$X = {}^3/_2$	$X = -{}^3/_2$
$X = 1$	$X = 1$	$X = -1$
◯	◯	◯

3. Put $4X^2 - 12X + 5 = 0$ into the quadratic formula?

$a4X^2 - b12X + c5 = 0$	$\dfrac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(5)}}{2(4)}$
◯	◯

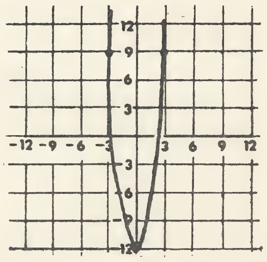
4. What are the roots of problem 3?

$X = 5$	$X = -5$	$X = {}^5/_2$
$X = 2$	$X = -2$	$X = {}^1/_2$
◯	◯	◯

5. Equations which have a second-order term will graph as what kind of lines?

curved	circular	linear
◯	◯	◯

6. Here is a graph of a parabola. What would the value of X be if Y = 9?



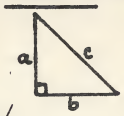
$X = 2$	$X = -2$	$X = -3$
◯	◯	◯

1. Which of these is a right triangle?

2. How many total degrees are there in a triangle?
   - 180°
   - 90°
   - 270°
   - 360°

3. What is the side opposite the right angle in a right triangle called?
   - arm
   - hypotenuse
   - leg

4. What is the <u>cosine</u> of this triangle?

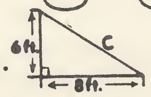


   - a/c
   - c/a
   - b/c
   - c/b

5. What is the <u>sine</u> of the triangle in problem 4?
   - c/a
   - b/c
   - a/c
   - c/b

6. What is the <u>tangent</u> of the triangle in problem 4?
   - a/c
   - a/b
   - b/a
   - b/c

7. Find the length of c.



   - 10 ft.
   - 14 ft.
   - 100 ft.

BASIC ALGEBRA Reference Folder Ma 1

The Logic of Algebra

1. This series of programs is about a branch of mathematics called algebra. Like any subject, it can be made difficult to study, and there are some advanced parts of algebra that are hard to understand until you learn the basics. But the things you need to know about algebra for ordinary use are easy to learn. This is what we will cover in this course.

2. Algebra uses a group of methods to help you solve problems about the relations between things which may be a bit too complicated just to keep in your head. You can write these relations down, and arrange them so that you can solve the problem; or at least you can show the relationships in the simplest possible form.

3. Suppose your cousin Charles is 24 years old. You could use the letter C for his age, and write C=24. This doesn't help us very much, since this is an identity. C and 24 are the same thing, like 5=5, but if we didn't know C is 24, the use of the letter C for his unknown age could be helpful; we'll show you why.

4. Suppose you remember that three years ago Charles had his 21st birthday. How old is he now? A simple equation would be C=21+3, and we would easily solve this equation for C. How old is Charles now? (3 years old) (21 years old) (24 years old)

5. Of course, 24. But suppose Charles was 10 years old when his sister Betty was born. How old is Betty now? Her age is 10 years less than Charles. We could say that B=C-10, using B for Betty's age. And if C is 24, then B=24-10, which makes B=14. How old is Betty? (10 years old) (14 years old) (24 years old)

6. Correct. These problems are of course quite simple, but you can imagine that equations in algebra may be used to solve quite complex problems. In algebra, we use letters and other symbols to stand for an unknown quantity. In algebra, what would you mean by B or C, or X or Y? (John's age) (13 years) (unknown quantities)

7. Yes. In algebra we could add, subtract, multiply and divide expressions with these symbols just as in arithmetic. We should be careful about one matter, however; for since the letter X is often used in algebra for an unknown quantity, we can't use it any longer as an arithmetic sign for multiplying. Instead, we may use a space and a raised dot between numbers; or we can use brackets, or parentheses. Thus, 2 dot 3 means two times three, and so does 2 next to a 3 in parentheses.

8. What is the value of this expression? $4 \cdot 5$ (9) (20) (45)

9. When you multiply numbers which are represented by letters or symbols, you don't need a raised dot, or parentheses, to show multiplication. For example, you may write XY=6, and if Y stands for 3, you know that X is 2. What does πB stand for? (3.1416 times B) (+B) (Pablum)

10. Yes. Letters which represent unknown or variable numbers are shown just like arithmetic numbers in algebra when you add, subtract or divide, except that generally we avoid the division symbol with a bar and a dot above and below it. Instead, we use the horizontal line, which is used for fractions to separate numerator and denominator. The expression on the top is divided by the term or expression below the line.

11. Here's a hint which may save you some trouble: Try to avoid expressing a division operation by saying "divide 3 into 27" or "divide Y into X." It's always less confusing to say "27 divided by 3" or "X divided by Y" or even "27 over 3", or "X over Y."

12. In algebra, brackets or parentheses allow you, temporarily, to treat an expression with more than one term just as if it had only one term. For example, X+3 can be enclosed in parentheses, and then multiplied by a number or letter as if it were one term. How could you show the quantity X+3 multiplied by Y? (XY+3) (X+3Y) [(X+3)Y]

13. Right. Show how the product of 'A' times the quantity B+2, would look. (A·B+2) [A(B+2)]

14. Right. Another way to show the product of 'A' times the sum of B plus 2 would be to multiply 'A' times each of the terms; that is, 'A' times 'B', then plus 'A' times 2. This would then give us AB+2A. Notice that for convenience we arrange the letters which are multiplied in alphabetical order. Also, to avoid confusion, we like to put numbers first, in product terms.

15. One law in algebra is the commutative law, which says that 3 plus 2 is the same as 2 plus 3, and 3 times 2 is the same as 2 times 3. This law works for addition and multiplication, but would it be correct to say that 3 minus 2 is the same as 2 minus 3; or 3 divided by 2 is equal to 2 divided by 3? (Yes) (No)

16. Right. The associative law of addition and multiplication tells us that when you add 2 plus 3, then add 4, you get the same answer as when you add 2 to the sum of 3 plus 4. Also you may multiply 2 by 3, then later multiply the product by 4, to get the same result as if you multiplied 2 times the product of 3 and 4. What's the name of this law? (assembly law) (associative law) (absolute law)

17. By these algebraic laws we know that (A+B) in parentheses, plus C outside them, is the same as A plus (B+C) inside parentheses. This means you can either add A and B first, then C; or add B and C, then A.

18. Also, A times the product of B times C is the same as the product of A times B times C. According to the associative law, it doesn't matter in which order you multiply. However, subtraction and division operations must stay in order. Which law allows addition or multiplication operations in any combination? (associative) (commutative)

19. There's another law of algebra called the distributive law, which concerns the combination of addition and multiplication, and is a bit harder to remember. A little practice will help, however. The distributive law tells us that 2 times the sum of 1 and 3 can be calculated either by multiplying 2 times 1, then 2 times 3, then adding the products; or by adding 1 and 3 and multiplying two times the sum. Here we perform both addition and multiplication. What's the name of this new law? (associative) (commutative) (distributive)

20. Right. If we multiply 2 times π times A times B, we would write it as 2πAB. This is called a single term. You can see there are no plus or minus, or any other arithmetic operation signs, because the grouping implies multiplication in algebra. When we write 2π over AB we mean that 2 and π are multiplied together, and then divided by the product of A and B. What would you call "3 times 4"? (a term) (a germ) (a binomial)

21. Yes. When an algebraic expression has more than one term, it is called a polynomial. There is a special word used for a polynomial with just two terms, like 2X+3Y. What would you guess it is called? (binomial) (bicycle) (trinomial)

22. Of course. You remember that we use the parentheses to enclose several terms so they can be treated as one

quantity; like (B+2) inside parentheses. When we wish to show that this binomial, B+2, is multiplied by something else, we use the parentheses, and indicate, for example, A times (B+2), by writing A parentheses B+2. We could remove the parentheses by multiplying A times each term in the binomial. In this case we would have AB+2A. Look this over carefully before you push the center button to go on.

23. Let's say that we were looking at a family album and Betty showed us a picture of Charles and herself taken a few years before. At that time, Betty said, she was half as old as Charles. If B is Betty's age when the picture was taken, and C stands for Charles' age at the time, what would be a true equation statement? (B+C=2) (C-B=2) (B=½C)

24. Right. We could also say that Charles' age was twice Betty's, or C=2B. Let's write another equation about Charles' and Betty's ages. Charles is 10 years older than Betty. How would you say this in algebra? (C=10B) (C=B+10) (C=B-10)

25. Correct. But before we go ahead, we'd better remember that although Betty will always be 10 years younger than Charles, there was only one time when she was half as old as Charles. You can probably guess that this was when she was 10 and Charles was 20 years old, so perhaps you don't need algebra to find when the snapshot was taken. But what if we found that Betty was 7/12 as old as Charles when he went to work at the post office? Algebra would help to find their ages easily.

26. If one morning you sold a customer 2 pounds of apples and 3 pounds of carrots, another customer 4 pounds of apples and 1 pound of bananas, and a third customer 2 pounds of bananas and 5 pounds of carrots, how much did you sell altogether? You could say 2A plus 3C for the first customer's apples and carrots, then 4A and 1B for the second customer, then 2B and 5C and so on. Obviously you could combine the same kinds of fruit and vegtables for your total sales that morning. If you didn't mix apples and bananas, what would you have as an algebraic sum? (6A+3B+8C) (9AB+24BC)

27. If you found five walnuts and seven pecans in the woods, and gave your friend three pecans and two walnuts, what would you have left? Let X stand for walnuts and Y for pecans. (8X+9Y) (3X+4Y)

28. Right. You must be sure to add or subtract only "like terms." You remember a "term" in algebra is just one letter or number, or if there is more than one letter or number, they are all multiplied together. There are no addition or subtraction operations within an algebraic term. "Like terms" are those in which all the letters are the same, even though the numbers may be different. 2AB and 3.6B are like terms. 4XY and πXY are like terms, since π is a fixed number. But you can't add 4XY and 2X+3Y and then combine any terms. Which of these is correct? (4X+2X=6X) (4X+2Y=6XY)

29. If you wished to know the number of square inches in a floor tile 10 inches long by 12 inches wide, you'd multiply 10 by 12 and get 120 square inches. If you had a number of tiles, some 9 inches square, and some 7 inches square, you'd have some tiles which covered 81 square inches, and others which covered 49 square inches. Any number multiplied by itself can be represented by the number with a small 2 at its top right.

30. Thus, X times X is called X "squared" and is written like this: X^2. X times X times X is called X "cubed" or sometimes "X to the third power." In algebraic terms it would be awkward to write 2XXXX, so we say "2X to the fourth power", or "2X to the fourth." These small numbers at the upper right are called exponents.

31. You can see, however, that 3X and $2X^2$ are not the same kind of terms, so they can't be combined

by addition to give 5X or $5X^2$. They can be added, of course, but only as separate terms. What would their sum look like? ($5X+5X^2$) ($5XX^2$) ($2X^2+3X$)

32. Right. What would you call $2X^2+3X$? (a term) (a binomial) (a mixture)

33. You remember that when you multiply 2 times 3 to get 6, we call 6 their product. X times Y gives the product XY. X and Y are called the factors of the term XY. In the term $3XY^2$, 3, X, and Y are the factors, and the factor Y of course occurs twice. What are the factors of $7AB^3$? (3, 7, A and B) (7 and 3) (7, A, and B (3 times)

34. How would you show the sum of 2X, and 4X+3? (2X+4X+3) (9X) (6X+3)

35. How would you show the sum of X+2, and Y+3? (XY+5) (X+Y+2+5) (X+Y+5)

36. How would you show the product of 2, and 3Y? Remember, a product is the result of multiplication. (2+3Y) (6Y) (Y+6)

37. How would you show the product of 2A and 3B? (2A+3B) (5AB) (6AB)

38. What's the product of 3, times the binomial 2X+3Y? (6X-9Y) (6X+9Y)

39. What's the product of 5 times the binomial 2X+3Y? (7XY) (10X+15Y)

40. In algebra, what does X usually represent? (some unknown or varying amount) (the operation of multiplication)

BASIC ALGEBRA

Algebra

Reference Folder Ma 2

1. This is your second program about algebra. You will recall that algebra is the branch of mathematics which helps us solve problems about complex but logical relationships among quantities. Algebra is like the science of logic, which deals with the non-mathematical relationships between things, or between events.

2. If you are clever, and have a good memory, and notice everything which occurs in a situation, you may be able to solve problems about some moderately complex relations between quantities without bothering to use the procedures of algebra. We do this every day with simple relations between quantities such as computing the miles we drive per gallon of gasoline. But as the relationships get more complicated most of us need to use the procedures of algebra, if we are going to find our own answers to the problems.

3. Some persons don't develop or sustain the habit of thinking logically about matters which arise in their lives, at work or at home. They act impulsively or emotionally, without bothering to think about the results. But when complicated matters arise, it is helpful to be able to find some logical and effective way to think about them. The study of algebra will help you do this.

4. In the first algebra program we used letters or other symbols to stand for unknown quantities, and learned some of the ways these quantities can be expressed. The most useful expression in algebra is an equation, which is a statement that one or more terms on the left side of an "equal" sign is equal in value to one or more terms on the right side of the sign. How could you define an equation? (how to ride horses) (statement about equal amounts) (diameter of the earth)

5. How might you describe algebra? (system for logically solving problems about related quantities) (study of relating mysterious things to the unknown)

6. Yes. We have already used the word "term," in algebra, as made up of a letter or number, or several letters or numbers, which are multiplied together. 2XY is a "term." Usually, however, we call it a term only when it is part of an algebraic expression made up of 2 or more terms added together. In the binomial 2XY-3Y, 2XY is the first "term." What is the second term? (2XY) (-3Y) (3Y)

7. Yes. You recall a binomial is a polynomial with just two terms. Let's make up a simple algebra problem. Two cans of paint, which have an unknown weight are combined with 2 one-pound weights; they are found to equal a 10 pound weight. Which shows this? (2X+2=10) (4X=10)

8. Yes! Of course you probably could find the solution to this problem easily without algebra, but let's go through it for practice. The first thing we must remember is that the balance of the equation must not be disturbed. We may add a pound or two to each side, or subtract it, but we must always remember to do the same thing to each side, or the statement about equality (which makes it an equation) is destroyed.

9. To solve an equation, or find its solution, means to find the value of the unknown quantities. An equation is solved when the letter representing the unknown appears by itself on one side of the equation, and some specific quantity appears on the other side. X=3 or X=5 would be statements which solved some sort of equation, since X would appear by itself. In the case of our equation 2X+2=10,it appears by itself. In the case of our equation 2X+2=10,it appears reasonable in simplifying it to subtract 2

pounds from both sides. What would we get then? (2X+4=12) (2X=8) (X-1=10)

10. Yes. Now we must take just one more step to find the solution. It's obvious that by dividing both sides of the equation by 2, we'll have the unknown, X, alone on the left side, which we've said means "solving it." What will we have on the right side? (X=4) (X=16)

11. Right. The equation X=4 is the "solution" of our original equation, and the number "4" is called the "root" of the equation. In some solutions, as we will find in more complex equations, there may be more than one "root," or correct answer.

12. It hardly seems worth employing algebra to solve problems like "if four X equals 24, what is X?" or "if X plus 5 is 15, what is X?" But starting with these easy ones will help us grasp the concepts which can later be applied to the more complicated problems.

13. A supermarket has a cash register which returns your change automatically. If you gave the cashier a dollar bill for purchases totaling 57 cents, including tax, and your change came to you in pennies and nickels only, how many nickels would you get? Let n be the number of nickels and p be pennies; then n+p would equal the number of coins. Multiplying n by 5 and p by 1 gives the value of the change received. This expression equals 100-57 shown on the right side of the equation. Could you use this equation to help you find the number of nickels? 5N+P=100-57 (Yes) (No)

14. Yes. First, let's subtract your purchases and see that 5N+P equals 43 cents change. Now let's assume that we were given the smallest number of pennies possible, which would be 3. These 3 pennies are worth three cents, so we may replace P, the value of the pennies by 3 and have 5N+3=43. Now to get 5N by itself, what must we do next? (divide both sides by 43) (add 5N to both sides) (subtract 3 from both sides)

15. Right. This would give "5N equals 40." To get N by itself, what do we do now? (divide both sides by 5) (add 40 to both sides) (punt)

16. Yes. When we divide both sides by 5, we get "N equals 8," meaning we got 8 nickels back from the cash register. This may seem to be a rather silly way to use algebra, but you can see that it can be a powerful way to solve more complex problems; finally even how to go to the moon or Mars.

17. Some equations are general statements about things which vary in value or amount, and are called variables. How would you define a variable? (something very terrible) (something which varies in value) (something very old)

18. Right. The temperature of the air outdoors is certainly a variable. The speed of your car when you drive it is a variable. These variables may be related to other variables by rules in the form of equations. These rules in equation form are called formulas. We all know that the area of a rectangle is given by A=LW, where area is A, and L and W are length and width; similarly $A=S^2$ for a square with four sides, S; and $A=\pi r^2$ for a circle, and so on.

19. Here is an approximate formula for converting your speed in miles per hour to kilometers per hour. K=1.6M. This means that if your car's speedometer read 10 miles per hour you would be traveling at a rate of 16 kilometers per hour. If you were driving in Mexico, and if you measured 50 miles per hour, it would be approximately 80 kilometers per hour.

20. You can see that a formula equation like K=1.6M is easy to use. You could make up a table or chart for

various values. You could change the way the formula is written to $\frac{K}{1.6}$=M, or .625K=M, or even 5K=8M by dividing or multiplying both sides by the same number, or by converting decimal fractions to common fractions, and so on. But there's not much algebra involved in this formula.

21. Let's look at another formula you might have to use if you went to a European country, for instance. Your Fahrenheit thermometer may read 50 degrees on a cool morning but the local centigrade thermometers show only 10 degrees! And in the afternoon when your thermometer rises to 68 degrees "F" the centigrade reading is only 20 degrees! This is certainly not just a plain ratio, like meters and miles; there's some adding or subtracting to do, as well as multiplying or dividing. Now algebra seems a little more useful.

22. The formula, as you may recall, is $F=\frac{9}{5}C+32$. Or we could solve for C and get $C=\frac{5}{9}(F-32)$. In this case you need to remember that the operation inside the parentheses must be done first; you can't just multiply the $\frac{5}{9}$ths by F, or by 32, and not the other inside term. What do you think a centigrade thermometer would read when the Fahrenheit temperature was 59 degrees? (5.9° C) (15° C) (27° C)

23. Yes. If you noticed that a thermometer sign by the beach on the French Riviera read 25 degrees, you'd remember $F=\frac{9}{5}C+32$. $\frac{9}{5}$ of 25, of course is 45. On the Fahrenheit scale, how warm is it at the beach? (65° F) (77° F) (82° F)

24. Right. You also see a lot of air conditioning thermostats in Europe set near 25 degrees. The Centigrade-Fahrenheit formula, which involves both addition and multiplication, is certainly not the most complicated use for equation operations, of course, but it begins to demonstrate the usefulness of logical procedures to solve problems.

25. A moment ago we discussed variables. A variable might be the temperature, or speed, the number of persons in a room, and so on. At any given time a variable, of course, may have a certain value, but if you are interested in what happens when its value changes, you consider it a "variable."

26. Other quantities in a relationship you are considering don't change, at least not under the circumstances you are interested in. These quantities are called constants, and you may know their value, such as $\frac{9}{5}$ths, or 1.6, or 32 or π, so you write them down as a number, or at least a symbol like π which stands for a fixed number. What is a "constant"? (a changing quantity) (a fixed quantity)

27. Yes. There is another word you may meet in algebra, called coefficient. In the formula $C=\frac{5}{9}(F-32)$ the number $\frac{5}{9}$ is a coefficient. This just means it's a multiplier written next to some expression number, or variable. What's a coefficient? (skillful worker) (multiplier)

28. In the term 2X what is the 2? (exponent) (variable) (coefficient)

29. Yes. The formula for the area of a square is $A=S^2$, where S is the length of the four equal sides. What do we call the little 2? (an exponent) (a variable) (a coefficient)

30. Of course. Maybe you remember all these words, but most students remember them better by repeating lessons like this several times. After all, how many times did you see that Alka-Seltzer commercial, or even that episode on "Dragnet"? It won't hurt to go through it again.

31. In the term 2X, what is the X? (variable) (constant) (exponent)

32. Where K stands for kilometers and M for miles, what kind of an equation is 5K=8M? (French)

(binomial) (Formula)

33. Yes. What do we call an expression like 2X+32? (single term) (binomial) (unknown)

34. Right. When we have a binomial in the numerator or denominator of a fraction, or a binomial enclosed in parentheses, we must always perform the addition or subtraction before multiplying or dividing by outside coefficients. We'll practice this again in our next program.

35. If we have reduced an equation to something like X=77, what is this called? (a problem) (an expression) (the solution)

36. Yes. In the solution equation X=77, what do mathematicians call the number 77? (the root) (the trunk) (the branch)

37. In the binomial 2XY+2X, what is 2XY called? (a term) (a semester)

38. Right. Another word used in mathematics is set. In ordinary use, this word means a collection of objects—a set of chess pieces, a set of dishes, or a set of golf clubs. In mathematics, also a set is a collection of things. The things in a mathematical set are called elements or members.

39. In mathematics, what would you call a collection of elements? (a group of chemicals) (a set)

40. What branch of mathematics deals with relationships between quantities? (geometry) (algebra) (alimony)

Dorsett Educational Systems, Inc. Box 1226 Norman, Oklahoma 73070 (405) 288-2300

BASIC ALGEBRA Reference Folder Ma 3

Solving Equations

1. You will recall that in the first two programs you have learned the definition of a number of words used in algebra: equation, term, coefficient, binomial, polynomial, solution, root and so on. You should review these two programs as often as necessary to be sure you are familiar with them and their meaning.

2. A "root" is the value of the unknown, while the "solution" is a form of the equation with the unknown (X, for instance) on one side and its value on the other. Which of these is the "solution" of this equation? 2X+3=7 (2) (2X=4) (X=2)

3. Yes. X=2 is the solution, and 2 is the root. In this equation, what number could be considered a "coefficient"? 3(X+2)=9 (3) (2) (9)

4. Right. The coefficient was a multiplier of the expression X+2 in the parentheses. What is (X+2)? (a term) (a binomial) (a solution)

5. Yes. And a binomial is a polynomial of just two terms added together or subtracted. Thus, X-2 is a binomial, also. You remember that subtraction is the opposite, or inverse, of addition, just as division is the opposite, or inverse, of multiplication.

6. Addition is certainly not the same as multiplication, of course, so subtraction isn't the same as division, but it is easy to remember addition-subtraction, and multiplication-division, as the two pairs of inverse operations which are used to solve equations. In the equation X+1=2, you have 1 added to X on the left side, so the inverse operation, subtraction of 1, to both sides, will solve it.

7. In the equation 2X=6, you have X multiplied by 2. The solution requires an inverse operation to this multiplication, so what should we do to both sides of the equation? (divide by 2) (multiply by 6) (subtract 2)

8. How would you solve X+5-8, and what is the solution? [add 8 (to both sides), X=13] [subtract 5 (from both sides), X=3]

9. Yes. How would you solve 4X=12, and what is the solution? (divide both sides by 4, X=3) (subtract 4 from both sides, X=8)

10. Right. How would you solve 6X=3? (divide both sides by 6, X=$\frac{1}{2}$) (multiply both sides by 6, X=18)

11. Yes. In this case the root was a fraction, 3 divided by 6, which is the same as $\frac{1}{2}$.

12. How would you solve the equation X-5=10? [add 5 (to both sides), X=15] [subtract 5 (from both sides), X=5]

13. Right. You get rid of the minus number combined with X by adding an equal plus number to both sides. How would you solve X-3=2? (subtract 2, X=1) (add 3, X=5)

14. Yes. How would you solve 17X=34? (divide both sides by 17, X=2) (subtract 17 from both sides,X-17)

15. How would you solve $\frac{1}{3}X=4$? (divide both sides by $\frac{1}{3}$, X=12) (divide both sides by 3, $X=\frac{4}{3}$)

16. Right. Another way of writing $\frac{1}{3}X$ is $\frac{X}{3}$. This is because the multiplier "one" is always assumed, and may not always appear. How would you solve $\frac{X}{3}=4$? (multiply both sides by 3, X=12) (divide both sides by X, $X=\frac{3}{4}$)

17. O.K. Now solve $\frac{X}{5}=6$. (divide both sides by 5, $X=\frac{5}{6}$) (multiply both sides by 5, X=30)

18. Right. Solve X-153=2. (add 153 to both sides, X=155) (divide both sides by 2, $X=76\frac{1}{2}$)

19. Yes. Now solve $\frac{X}{2}=36$. (divide both sides by 2, X=18) (multiply both sides by 2, X=72)

20. Yes, of course. By the way, you might have the X in the denominator, or on the bottom of a fraction as $\frac{6}{X}=2$. What do you suppose you should do then? (multiply both sides by X) (punt)

21. Yes, this would then give you 6 on the left side of the equation equals 2X on the right side. You shouldn't feel uncomfortable, incidently, with the unknown on the right side. If you do, you may rewrite it in reverse. But be sure to reverse it carefully, for in a long equation with a lot of operation symbols you might make an error as to where the equal sign occurred. For now, we can say 6=2X or 2X=6, as you prefer. In any case, it appears that we need another step to solve it. What should we do now? (multiply both sides by 2, X=12) (divide both sides by 2, X=3)

22. Yes. You can see that in a simple division-type equation it is only a single step to solution, with the important exception of the case when the unknown is in the denominator. In that case, there is at least one extra step required. This is to multiply both sides by the unknown to put it on top, in the numerator position.

23. In solving equations, we must always remember to keep aware of the collection symbols of parentheses and brackets. Otherwise we may make the mistake of multiplying a coefficient just by the first term of a polynomial which should be collected within parentheses. The coefficient 2, when multiplied by the binomial (X+3) in parentheses, is 2X+6; if we had accidentally omitted the parentheses we might have just obtained 2X+3.

24. What is the product of 5 and the quantity (2Y+4)? (10Y+20) (10Y+4)

25. Right. Sometimes parentheses are used to separate arithmetic numbers which are to be multiplied together in a single term, instead of raised dots or the X-shaped "times" symbol used in arithmetic. Instead of 13 times 8 times 25 with X-shaped symbols, or with a raised dot, you may use parentheses. If you used the **X** multiplier symbol in an algebra equation, it might be confused with an unknown. If you use the dot, sometimes it gets confused with a decimal. And if you don't use parentheses either, what number could you mistake it for? (13,825) (1,385)

26. Yes. How would you solve 2X=(3+5)? (a. add 3 and 5, get 8; b. divide both sides by 2; X=4) (a. divide 3 by 2; $X=6\frac{1}{2}$)

27. Right. You perform the indicated operations on each side, then you go through the regular solution procedure to isolate the unknown.

28. How would you solve 3X=4(2+1)? (a. multiply 4(2), get 8; b. add 1 to 8, get 9; c. divide 9 by 3; X=3) (a. add 2 and 1, get 3; b. multiply 3 by 4, get 12; c. divide both sides by 3; X=4)

29. Yes; you should remember that a coefficient outside parentheses containing a binomial is multiplied by both

terms. You should do this, if both terms are arithmetic numbers, by adding or subtracting them as indicated, then multiplying the result by the outside coefficient.

30. How would you solve $5X=\frac{13+17}{3}$? (a. add 13 and 17, get 30; b. divide 30 by 3, get 10; c. divide both sides by 5, X=2) (a. divide both sides by 5, get $X=\frac{13+17}{15}$)

31. Right. Now solve: 3+4=7X. (X=7) (X=1)

32. Yes. Solve 3=7X-4. (X=7) (X=1)

33. Right. Actually the last two equations were the same thing: 3+4=7X and 3=7X-4 were different only because in the second form, 4 had been subtracted from both sides, and had to be added back to both sides, as the first step in the solution.

34. Solve this equation: $3(X-1)=\frac{6+2}{4}$. (a. subtract 1 from 3; get 2; b. divide both sides by 2; X=7) (a. add 6+2, get 8; b. divide 8 by 4, get 2 on right side; c. multiply X-1 by 3, get 3X+3 on left side; d. add 3 to both sides, 3X=5; e. divide both sides by 3; $X=\frac{5}{3}$)

35. Right! How about that! But it's not really hard if you just perform the indicated steps, on each side of the equation one at a time. First, you do the indicated addition, or subtraction, of any like terms inside a polynomial; then the indicated multiplication or division, if any. By this time you should be ready to isolate the unknown by inverse operations on both sides of the equation.

36. "Remember: First, perform the indicated operations of like terms on each side. Secondly, perform the inverse operations to isolate the unknown on both sides. In each case, do the addition-subtraction operations before the multiplication-division operations." Now look at these statements carefully because you are going to be asked some questions about them. Push the center button when you are ready to go on.

37. Which do you do first in solving an equation with like terms in a polynomial? (perform indicated addition-subtraction then indicated multiplication-division on each side) (perform inverse operation to both sides to try to isolate the polynomial)

38. Which is it better to do first? (addition-subtraction of like terms, if any) (multiplication-division)

39. Solve this equation: 3X=8+4. (X=3) (X=4) (X=5)

40. Solve this equation: $2X+3=\frac{15}{3}$. (X=1) (X=2) ($X=\frac{3}{5}$)

41. Don't forget to repeat this program as often as you need to, and the other lessons, too. Also, it will help if you will stop here and study some of the other mathematics programs in the Mf series to get familiar again with reducing fractions, grouping, and mixed numbers; and the Mg series to regain facility with squares, square roots, exponents and areas. Study the Mm series about graphs and charts, and the Mr series to get familiar again with ways of solving story problems.

BASIC ALGEBRA

Reference Folder Ma 4

Operations With Plus And Minus

1. In algebra you will be working with several different kinds of quantities. Some of these you have already learned about in your other math programs, especially in the Mp, Mm and Mr series. For example, there are real and imaginary numbers, rational and irrational numbers, 0 and non-0 numbers, and positive and negative numbers. In algebra negative numbers are quite common, and if you have a bank account, or have done bookkeeping, you may already have dealt with negative quantities.

2. A minus sign before a number in arithmetic always tells you to subtract. But in algebra you may also divide, add, or multiply a number with a minus sign in front of it because in algebra that simply means it's a special kind of number, and not just an arithmetic operation being performed. +5, +23, and 96 are all positive numbers; if a number doesn't have a plus sign before it, it is understood to be a positive number. But all negative numbers, when written, have a minus sign before them.

3. When we just talk about quantities, however, often we don't say "minus" or "negative," so you must be careful to write a number as negative when it really is. If a football half-back has gained 50 yards in the first half of the game, he could gain a negative 5 yards on the next play, if he's tackled behind the line.

4. In this case adding a negative gain would really be subtracting to get 45 yards total gain. This would seem to be indicated anyway, if you wrote it down "minus 5 yards." But, for example, if this halfback already had a net gain of minus 10 yards for the game so far, then you'd add the minus 5 yards to get minus 15 yards total. Now you know why they call it algebraic addition. You have to watch for the negative numbers. What would you get if you added +3 and -5? (+8) (+2) (-2)

5. Yes. If you deposited $10 into the bank, and you already had a balance of $50 in your account, you'd then have $60 total. If you wrote a $20 check, you could call it a "minus $20" entry added to your account. How much would your balance be then? ($30) ($40) ($50)

6. Right. Actually, it's not too difficult to work with negative numbers, but you must be careful. If your bank account were overdrawn $10, and you wrote a $2 check which the bank paid, how would you describe your bank balance? ($8 net) (-$12) (just fine)

7. Right. If you had a tank of brine, or salt water, and a thermometer reading in centigrade degrees, suppose you read +5 degrees. If you added some colder brine and the temperature dropped 10 degrees, the thermometer would read -5 degrees. The top of a mountain 10,000 feet above sea level might be considered 10,000 feet; near the Dead Sea it is about -1000 feet, or 1000 feet below sea level.

8. If you received two checks of $10 and $15 respectively in the mail, and deposited them in the bank, then wrote a check for $22, what was the net effect on your bank balance? (+$47) (+$3) (-$22)

9. My airplane cruises at 175 miles per hour, but yesterday I had a 60 mile-per-hour head wind. How fast was I going en route to Dallas? (175 mph) (115 mph) (60 mph)

10. You play "put and take" for pennies. You made these scores: Put 3, Take 6; Put 1, Take 4; Put 5, Take 2. How much are you ahead of the game? (3 cents) (2 cents) (1 cent)

11. Right. The rule about adding algebraically is to "add all the positive numbers together, if more than one, add the negative numbers, then subtract the smaller from the larger, and give the result the sign of the larger amount." Repeat this to yourself a couple of times before you push the middle button to proceed.

12. Let's add +3, -2, +4, and -1. First, you remember, we must add all the positive numbers together, then add the negative numbers. We get +7 and -3 as the respective sums. Then what must we do? (subtract 3 from 7, result is +4) (add 7 and 3, result is 10)

13. Yes. The sign of the result takes the sign of the larger number. Let's add +5, -7, -4, and +3. What's the result? (+3) (-19) (-3)

14. Right. Now, what's the first step in adding algebraically? (add positive numbers, add negative numbers) (find common algebraic factor)

15. The rule for subtracting algebraically is "change the sign of the subtrahend, which is the number being subtracted from the other, then add algebraically." To subtract a positive 3 from a positive 5, you add -3 to +5 and get +2. To subtract a negative 3 from +5, you must add +3 to +5, to get +8. You change the sign of what quantity in algebraic subtraction before adding? (minuend) (subtrahend) (living end)

16. Yes. Now subtract -7 from -10. -7 is the subtrahend, so you change it to +7. Now add +7 to -10. What is the result? (-17) (+3) (-3)

17. Right. What is the result of +13 after subtracting -5? (+8) (+18)

18. Right. Now subtract +6 from -8. (-2) (+14) (-14)

19. Yes. Now subtract +9 from +4. Remember, change the sign of the subtrahend, then add algebraically. (+5) (-5) (+13)

20. Yes. In adding algebraically the minus sign gives you a fairly clear idea of the proper operation, but multiplying and dividing negative numbers may be confusing for a while. One way to learn how is to start by this rule: "the product of two numbers with the same sign, is positive; and the product of two numbers with different signs is negative." +3 times +7 is +21; -2 times -5 is +10; -4 times -6 is +24; but +2 times -3 is -6! and -5 times +7 is minus 35 and so on. What's the product of +1 and minus 2? (-3) (+2) (-2)

21. What is the product of -3 and -8? (-11) (-24) (+24)

22. What is the product of +12 and -2? (-24) (+24)

23. What is the product of -9 and +4? (-36) (-24) (+36)

24. In dividing, you use the same rule as multiplying, since the inverse operational relationship of multiplying and dividing doesn't inherently involve the insertion of a minus sign, like subtracting. We know that +2 times +2 is plus 4, and +3 times -4 is minus 12, and -5 times -6 is +30. Now if +3 times -4 is minus 12, wouldn't you expect that +3 times $-\frac{1}{4}$ would be $-\frac{3}{4}$? And -5 times $-\frac{1}{6}$ is $+\frac{5}{6}$. Since division by 6 is the same as multiplication by $\frac{1}{6}$, you can see the sign of the quotient is affected by the same rule as the sign of the product. What is this rule? (when multiplying (or dividing) by like signs, result is positive; by different signs, it's negative) (when you accentuate the positive, eliminate the negative)

25. Right. What is the quotient of +21 divided by -7? (+3) (+14) (-3)

26. What is the quotient of +21X divided by -7X? (+3) (-3)

27. Yes. We can divide like terms, which means terms which are all arithmetic numbers, or contain the exact unknown to the same power. X divided by X is just one, 2X divided by X is just 2, 3X divided by minus X is minus 3. What is 2X times X? (2) (2X) ($2X^2$)

28. Right. 2X plus 3X is just 5X, just as 2 dozen eggs plus 3 dozen eggs is 5 dozen eggs. But if you found an old Egyptian flagstone that was 3 cubits wide and 5 cubits long, it would have an area of 15 square cubits. Similarly, if you multiply 4X by 6X you have $24X^2$.

29. Add +5, -1, +2X, and -4X. Don't forget you can combine only "like terms." (4-2X) (2) (2X)

30. Right. Add -3, +2X, and -5Y. (-6XY) (2X-5Y-3)

31. Yes. None of the terms were "like terms," so we just had to show them as a polynomial. Now add +4, $+2X^2$, and -8X. ($4-6X^2$) ($2X^2-8X+4$)

32. Correct. Like terms have to have the unknowns of the same exponent, or the same product of unknowns. You can't combine during addition or subtraction such terms as 2X, $3X^2$, 5XY, $4X^2Y$, $6XY^2$, and so on.

33. You remember from the previous program that the small exponent 2 in X^2 indicates the product of X times X, and X cubed means X times X times X, and so on. What does X to the fifth power mean; that is, X with the exponent 5? (X times X times X times X) (X times X times X times X times X)

34. While exponents are the small numbers to the upper right of a letter representing an unknown, a numerical coefficient is the multiplier, such as the 3 in 3XY or 3B.

35. And when you have like terms, which are numbers only, or terms with unknowns with the same exponent, such as +5X and -2X, you can add the numerical coefficients algebraically when you add the numbers, and nothing is done to the unknown. The unknown X in this case could be the temperature variation in 5-degree gradations hotter or colder, or it could be the distance you walk per hour north or south.

36. You may have, or can borrow, a small electronic calculator like this one. Algebraic addition and subtraction is easy with it, but there are still some things you will have to remember. To add a series of signed numbers is simple; just press the button to enter the number digits from left to right as on an adding machine, then enter the sign of the number by pressing the button showing that sign. Remember, in this kind of electronic algebraic addition that the number goes ahead of its sign; although when you write it down, the sign is in front of the number!

37. Using an electronic calculator, how would you enter (24+18-5)? (push 2, 4, +, 1, 8, +, 5, -) (2, 4, +, 1, 8, -, 5)

38. Using a calculator you would combine the collection of numbers as like terms in a polynomial by algebraic addition before you would multiply by outside coefficients or other polynomials. For example to evaluate 5(22-7) you would enter 22, then a plus, because it is a positive number, then 7, followed by

minus, and only then would you press the 5 button followed by the times and equal button to get the whole result. This way of combining like terms eliminates the temporary storage (electronically or on a scratch pad) of the different products if you were to multiply the separate binomial terms first, then add their products.

39. Of course, when all of the terms of a polynomial aren't like terms, first you merely collect what you can, then multiply each term by the coefficient, or other multiplication term. For example, to multiply 4 times the quantity (+3, plus 2X, minus 1, plus 5X), you would collect the numbers plus 3, minus 1, and also combine plus 2X and plus 5X, before you multiplied these sums by 4. What would you get? (8+28X) (43+68X) (12-8+8X+20X)

40. Yes. Now multiply B times the polynomial 5+2C-1. What's the collected product? (5B+10C-B) (4B+2BC)

41. Right. What's the product of -2 times the binomial X+1? (2X+2) (2X-2) (-2X-2)

42. Right, you change the sign when you multiply by a negative number. What's the product of -3 times the polynomial A-B+C-C? (-3A+3B-3C+3D) (3A+B+C-D)

43. Yes. Don't forget that a missing plus sign is understood to be plus in a single term and coefficient. Now multiply X times 2+X+3Y-4X+1. Don't forget to collect your like terms. (2X+2X+3XY-8X+2) (3X-3X^2+3XY)

44. Right. Don't forget, you should repeat this lesson, and the earlier lessons, several times to be sure you understand and remember all of the rules of algebra you have learned.

BASIC ALGEBRA Reference Folder Ma 5

Working With Sets

1. In the previous programs in this series on algebra you have learned about basic principles which have for many years been useful in solving problems about unknowns. In addition to these classical principles of algebra, students in recent years have been studying some general principles for dealing logically with ideas. Part of these principles are included in what we know as the Theory of Sets.

2. In this program you will learn a few of the ideas employed in Set Theory, so that as you study more advanced ideas, you will know some of the language and terms. If you have studied the so-called "modern mathematics," you may already be familiar with many of them.

3. Although the theory of sets has been widely studied for only a short time, much of the early work on it was performed by George Boole, and others, a century ago. Boole, an Irish mathematician, worked out a system of logical operations, some of which applies to the design of today's electronic computers.

4. Based on what you've heard, how would you describe a thing called "Boolean Algebra"? (method of making logical operations and decisions) (French Alphabet Soup) (an alternate quadratic solution)

5. Yes. For the purposes of your study, let's say that a set is "any collection of things." Instead of "things" we may occasionally say "objects," although the things in a set could be intangibles. "Set" is a short, handy word we often use to describe the articles in collections–like silverware, dishes, chess pieces, and so on which go together. They have some common characteristics, but may not be exactly alike.

6. The things, or objects, in the set are called, in mathematics, the elements or members of the set. You might collect a set of numbers which you could use in a table, list, or directory. In this case the numbers are members of a set. If you made up the list of numbers whose values are obtained by applying a formula, how would you describe the list? (random list of collected numbers) (set of values which satisfy the formula)

7. Right. How would you describe a set, as used in mathematics and logic? (any collection of objects) (firm and hardened) (prepared, ready to go)

8. Yes. What do mathematicians call the things collected in the set? (pieces, parts) (components, collections) (elements, members)

9. Right. In general, a set may have any number of elements, although some sets, like a list of the basic names for days of the week, whether it was in French, German, or English, would have a definite number. A set of values satisfying a formula, however, might have any number of elements, even though just any numbers wouldn't do; they are specific numbers, of course.

10. Would you think there could be a set with no elements at all? (in math, yes) (perhaps, maybe) (never happens)

11. Yes. Perhaps at times you've even talked about such a set; in mathematics it can occur frequently. The set of airplanes which flew successfully in the nineteenth century has no elements or members;

such sets are called "empty sets" or "null sets." This tends to prove which of these statements? (all sets have a definite number of elements) (a set may have any number of elements)

12. Right. If you had a basketball club with a set of 20 members, and six of them were less than 5 feet tall, you might call this a "subset." Another subset, however, could be made up of two players over 7 feet tall. A subset, then is made up of members of a whole set, but has some extra characteristic.

13. A set, you know, may have any number of elements, from 0, up to the limits, if any, imposed by the character of the set. A subset, then, has a limit, of a sort. What is it? (no more than half of the basic set of elements) (no more than 5280 elements) (no more elements than the basic set)

14. Right. By definition all the elements of the subset must also be members of the basic set of elements. So a subset may have none, a few, or all of the elements of a set, but no more. The subset of the number of boys in the algebra class may be zero or all of the set of class members. But in the girls' gym class, it can't be less than zero; and on the boys' hockey team, it can't be more than the entire roster.

15. You already use parentheses and brackets like these at the top. These new symbols, the braces, have been used for years by typographers for showing the inclusion of things like rules or statements into a collection or set with a common feature. Mathematicians usually read these braces as "the set of."

16. Then how should you read this expression? $\{2, 4, 6, 8\}$ (2, 4, 6, 8 in braces) (the set of even integers)

17. Which of these is shown as the set of prime number integers? ($\{1, 2, 3, 5, 7\}$) (1, 2, 3, 5, 7)

18. Yes. If you apply a formula, such as $F^o = \frac{9}{5C^o} + 32$ for converting centigrade or Celsius degrees to Fahrenheit, you could get as many elements as you wish. But ordinary algebraic equations, like linear equations, may have just one member to the solution set of the equation.

19. You have studied quadratic equations. How many members are there to the solution set of a quadratic equation? (1) (2) (3)

20. Yes. What is the solution set of this equation? $X^2 - 7X + 12 = 0$ ($\{X = 3, X = 4\}$) ($\{X^2 = 5\}$) ($\{X=12\}$)

21. Yes. As you learned, some sets have a very large, or even indefinite number of elements. You probably know that mathematicians call an indefinitely large number an infinite number. An infinite number is in some ways difficult to use, but we know, for example, that the set of all numbers greater than zero is indefinitely large.

22. The set of all numbers greater than a trillion is indefinitely large, for that matter, but mathematicians can deal with infinite numbers, or at least symbolize them (∞). For example they may describe a set by saying "the set of all X's such that X is greater than 5" by writing what you see here. $\{X | X > 5\}$ Which of the symbols means "the set of"? ($\{\ \}$) (|) (>)

23. Yes. And you can guess that, reading left to right, the vertical line means "such that" and the symbol like the typographer's angle bracket menas "greater than." How do you suppose you would symbolize "X is less than 6"? ($X > 6$) (Xi | t6) ($X < 6$)

24. Right. How would we express in words the description of this set? $\{X | X < 6\}$ (the set of all values of X such that X is less than 6) (X bar X; but it's greater than 6)

25. What are the members of this set? $\{X / X^2 = 4\}$ ($\{4,0\}$) ($\{2\}$) ($\{2, -2\}$)

26. You remember the distinction between a set and its subset. Sometimes we think of a set as being a part of a very large or Universal Set, sometimes symbolized, (as Boole did) with an "I" for infinite, but now more often with a capital U for "universal." It actually means just a large population, or all the members, the totality of the category being discussed, from which our basic set members are specified.

27. We might say that U is the set of all students in a school, and the set $X = \{F \mid F$ is a football team member$\}$. The universal set is often diagrammed as a large outer rectangle.

28. While we may write out the main distinguishing "properties" of a set, there are usually a number of properties which are implicitly understood. If you are describing a set of persons, for example, you would assume they are normal individuals from the planet Earth. From the universal set of all Philadelphia lawyers, for example, set L may be made up of P, Q, and R—the senior law partners of Parker, Quillan and Robb.

29. Sometimes two sets described by different properties may be found to be equal. The five best students in English may also be the five best students of mathematics. In this case we note that the sets are equal, as shown by an equal sign. ($E = \{h, i, j, k, l\}$ five best in English; $M = \{h, i, j, k, l\}$ five best in mathematics; $E = M$)

30. If set A = set B, even though the description of their properties is different, what can we say? (they contain exactly the same members) (they are related in some general way)

31. Yes. There is a symbol for a member-to-member relationship. If set C is a set of 15 members of a golf club, and set D is a set of their 15 lockers, we call these "equivalent" sets, and use a double-ended arrow. $C \longleftrightarrow D$

32. Venn diagrams were used in the nineteenth century by John Venn, an English scholar. In this diagram the rectangle enclosed a universal set; a circle A is a given set, and a smaller enclosed circle, B would be what? (an element) (a member) (a subset)

33. The sets C and D are equal, so here they are represented by a single circle.

34. Here are two overlapping sets, E and F. Some of the members of the local Lion's Club and some of the members of the Kiwanis, also work for the telephone company.

35. Disjoint sets are sets which don't have any of the same members. If you joined them, their union would be symbolized by a "cup" like this $\cup$, and the resulting set would have both properties. The number of members would be the sum of the number of members in each set. Would this be true of a union of overlapping sets? (Yes) (No)

36. Right. One or more of the elements was in the overlapping area, and wouldn't be counted twice. If T is a subset of S and they are joined by union, or "S cup T", would there be any increase in the number of elements in S? (Yes) (No)

37. Right. A subset can be created by the common area within the intersection of two sets. This intersection operation is shown as a "cap." If an equation or formula defines the properties of one set, and

another defines the other set, the intersection subset will contain the element or elements which do what? (satisfy both equations) (are eliminated from both circles)

38. Yes. Are there elements which satisfy the properties of each of two disjoint sets? (Yes) (No)

39. Which of these tables is correct?

$\leftrightarrow$ is "equal"	$\leftrightarrow$ is "equivalent"
$\cap$ is "cap" (intersection)	$\cup$ is "cup" (union)
$\cup$ is "cup" (union)	$\cup$ is "cap" (intersection)

10. Add these numbers: +9, -7X, -3, +5X, -8, +6. What's the total? ($+2X$) ($+2$) ($4-2X$)

11. Right. Let's continue with a little review. What's the exponent way of expressing $2X$ times $3X^2$ times $4X^3$ times $5X$? ($234X$) ($120X^7$) ($240X^3$)

12. Yes, you multiply numerical coefficients, and add exponents. X^7 times X^5 is X^{12}, and by the way, X^5 divided by X^2 is X^3, as you probably remembered.

13. What is the sum of -2, +3A, -7B, +1, -A, +4B, +6? ($7AB$) ($2A+3B-5$) ($2A-3B+5$)

14. What is the total of ($2X+4$) added to ($7-5Y$)? ($2X-5Y+11$) ($18XY$)

15. Yes. Multiply $-2A$ times ($3B+4C-5$). ($6AB-4AC+10A$) ($-6AB-8AC+10A$)

16. Right. Divide ($6X^2Y-4XY+2X-10$) by $2X$. ($3XY-2Y+1-\frac{5}{X}$) ($3XY-2Y+2X+\frac{1}{5}X$)

17. A company covers a counter top with one-foot-square tiles of formica, but measures the top size in inches. How would you express the counter area in square feet when length and width are read off in inches? ($A=12LW$) ($A=144LW$) ($A=(\frac{1}{12}L)(\frac{1}{12}W)$)

18. Yes. So much for review and practice. Now let's try something different and harder. How would you multiply ($2X^2+7+13$) times ($3X+5X+9$)? Well, to be frank, not easily. You can't do it by punching a few buttons on your electronic calculator, but it can be done by a straightforward way in which you multiply every term in the multiplier by every term in the other polynomial, then add all the like terms that result from these products. You are going to have one term to the fourth power of X, and a term without an X, and several product terms with X to the first, second and third powers which you can add respectively together.

19. Let's start with an easy one. What's ($X+2$) times ($X+3$)? Using the X term from the first binomial and multiplying it times $X+3$, we get X^2+3X. Then $+2$ times $X+3$ gives $2X+6$. Add X^2+3X and $2X+6$, and you get X^2+5X+6. The only like terms were $3X$ and $2X$, so the rest had to be listed separately. Now multiply ($X+1$) times ($X+4$). (X^2+5X+4) (X^2+5) (X^2+8X+4)

20. Right. That's easy. But how about $X+2$ times X minus 3? Use a scratch pad if you need to. (X^2+5X+6) (X^2+X^2-6) (X^2-X-6)

21. Yes. You must watch those algebraic additions, and with long polynomials it's necessary to line up the product terms in columns so that the algebraic sum of these like terms can be obtained without confusion.

22. Now how about ($X+3$) times ($2X+4$)? ($2X^2+10X+12$) ($2X^2+7X+12$)

23. Right. With a lot of tedious practice you certainly could get skilled at doing this sort of multiplication of polynomials by polynomials, but there really isn't much to it except being careful and thorough.

24. Dividing a polynomial by a polynomial is called "factoring." Actually it is also called factoring when you divide a polynomial by a monomial, such as dividing ($2X^2+4$) by $2X$ times ($X+2$), but the really tricky factoring involves polynomials as divisor, dividend and quotient. For example, would you think that X^2-9, divided by $X-3$, would give $X+3$ as a quotient? (Yes) (No)

25. That's right. But it isn't always easy to look at a polynomial and guess what its factors are. For instance, X+3 times X+3 gives X^2+6X+9; and X-3 times X-3 gives X^2-6X+9.

26. But, no doubt, soon we will have pocket algebra calculators, or "micromini" computers to do this, so we don't need to do much drill on dividing, or factoring. Just remember that when you see an equation with the unknown squared, and a minus sign on the "pure-number" term, but no term with the unknown at the first power, it can be factored easily. For example, $4Y^2$-25 gives 2Y+5 times 2Y-5. $9X^2$ has for factors 3X+6 times 3X-6. What are the factors of A^2-1? [(A+1)(A-1)] [(X-A)(X+A)] [(A-1)(A-1)]

27. Another common kind of polynomial occurs with a so-called "perfect square" when the coefficient of the unknown in both the factors is only 1. For example, X+1 times itself is X^2+2X+1; (X+2) squared is X^2+4X+4; X+3 squared is X^2+6X+9; X+4 squared is X^2+8X+16; and so on. What would X+5 times X+5 equal? ($5X^2$+10X+15) (X^2+10X+25)

28. Yes. Now guess what the binomial (X-6) times itself equals. Watch that minus sign! (X^2-36) X^2+12X-36) (X^2-12X+36)

29. Yes. You see, for example, that X-1 times X-1 gives X^2-2X+1, because the minus 1 times minus 1 gives the last +1 number term, but the two multiplications of -1 times the plus X's add up to -2X. If you were to drill on this kind of factoring you'd soon be able to recognize the trinomial products of squared binomials such as X-5 times X-5 is X^2-10X+25, and so on. What do you think is the product of 2X-7 times itself? ($4X^2$-28X+49) ($2X^2$+14X-49)

30. What is the product of (X+7) times (X-7)? Watch those signs! (X^2-14X-49) (X^2-49)

31. What are the factors of X^2+6X+9? [(X+6)(X+9)] [(X-3)(X+3)] [(X+3)(X+3)]

32. What is the product of (X+10) and (X-10)? (X^2-20X+100) (X^2-100)

33. What do you think is the product of X+3 and X+2? (X^2+5X+6) (X^2+6X+9) (X^2-6)

34. What's the product of (3X+1) and (3X+1)? ($6X^2$-1) ($9X^2$+1) ($9X^2$+6X+1)

35. Guess what the factors must be of $10X^2$+23X+12. [(5X+6)(5X+6)] [(2X+3)(5X+4)]

BASIC ALGEBRA

"AL-JEBR"

1. Algebra may seem, for a while, to be confusing and disturbing, but it is actually a way to straighten out and simplify relationships. The capable Arab mathematicians of more than 1000 years ago called it "al-jebr," meaning bonesetting, or straightening; so you can see they thought of it as a way to solve important problems, and you can think of it this way, too.

2. If you were only to deal with simple problems all your life, perhaps you'd never need algebra. But even making out a will, when someone wishes to leave assets divided in a certain way, depending again upon the heir's age, whether living, whether more heirs are born, and so on, through even a relatively few conditions required by the person making the will, you would like to have some system for expressing the various relationships and calculating the quantities. In other words, you need a method like algebra to straighten out relationships.

3. When John is 21, he will get an endowment. He will be twice as old as he is today. How old is he? He's X years old, and 2X=21, so when we solve the equation by dividing both sides by 2 we get what? (X=21) (X=19) (X=10½)

4. Mary was new at her job on Monday and only finished a few assembly parts, but Tuesday she did twice as many and Wednesday three times as many. The foreman came by Wednesday evening and noticed Mary had finished 24 assemblies total for the three days. How many did she complete on Monday? If Monday's work was X, then X plus 2X plus 3X equals 24. Solve for X. (3) (4) (5)

5. Right. You combined the like terms, then divided both sides by the coefficient of X, which was 6, and got the solution. We've been doing this sort of thing in the first four lessons, and will do some more in this one.

6. Before we go on, however, let's take a moment to remember that to get a solution, or solve an equation, means to arrange to isolate the unknown. You are interested in one side of the equation, generally the left side. This unknown, for a complete solution, should have no numerical coefficients, (except the understood coefficient of one, which is not written) and no exponents, (except 1, which again is not written) and no sign, (except plus, which is also usually not written). Sometimes the omission of the plus sign on an isolated term, or the first term of a polynomial, or the omission of the integer one as coefficient or exponent, can cause you to make a mistake, so you'll have to be careful.

7. This morning the highway repair boss said a fourth of the crew were out sick. You noticed that five men were missing. What's the size of the full crew? You could write ¼X=5 or ($\frac{X}{4}$=5) (it's the same thing.) X is the crew size. What's X? (X=5 men) (X=4 men) (X=20 men)

8. Yes. The sergeant reported, "Captain, sir, we lost four men on last night's patrol out of the 12 we sent out." If X were the men left effective in the patrol, how would you write an equation? (X+4=12) (X=12-4) (either is OK)

9. Karen had $28 left in her bank account after her last check for $5 had cleared. How much did she have before she wrote that last check? If it's X, then X-5=28. What would you do to solve this equation? (add 5 to each side; X=33) (subtract 5 from both sides; X=23) (divide by 5; X=5$\frac{3}{5}$)

BASIC ALGEBRA Reference Folder Ma 7

Algebraic Fractions

1. In algebra we deal with the logical relationships of known and unknown quantities. Sometimes there is just one unknown quantity, and it has some fixed value we can find out. Sometimes there's one unknown quantity, but we may find it may have several values, or can vary, like a speedometer or a chronometer readings, with different circumstances. Quite often there are two unknown quantities, like the ages of little Joe and his big brother Mike, but at a given time, with enough information, we can find them by algebra.

2. This year Mike is twice as old as Joe, who is 10 years old. We could say J=10 and M=2J, then write M=2 times (10) to get Mike's age as twenty. You could have known this much from such a simple relationship without algebra, of course, but you can imagine it would be harder to do in your head if we said that Mike is 15 years older than Joe was five years ago, when Joe was only a third as old as Mike. There a little algebra would be useful.

3. You probably can't see any point in tricky questions about Mike and Joe's ages, but it would be very important to calculate the relationship of a moon lander to its command module, or of the relationships or the arrival times and positions of 100 airline flights at Chicago's O'Hare airport between 5 and 7 o'clock. So if we practice on simple problems, it's just to learn the basis for solving the complex ones.

4. Mike and Joe flew their kites using up a 150-foot spool of string. Joe's string, he found, was only half as long as Mike's. How long was it? If Joe's string was J feet long, how long was Mike's string? (½J) (2X) (2J)

5. Yes. Joe's string was J feet long, and Mike's was 2J feet long. Together they used up all of the 150 feet of string. What is the correct equation for this problem? (J+2J=150) (J-2J=150) (2J=150)

6. Right. We collect the two like terms on the left side of the equation to get 3J=150; then we divide both sides of the equation by 3 to isolate the unknown, and get J=50 feet. Mike's kite string, being twice as long, used up the other 100 feet of the 150-foot spool.

7. If Mike always got twice as much string as Joe, no matter how long the spool was, we might say that M=2J. Therefore, if Mike's string were 150 feet long, how long would Joe's string be? (50) (75) (100)

8. If Joe took the first 50 feet of string, then a third of the string left on the spool, how would we write the equation? ($J=50+\frac{1}{2}M$) ($J=50+\frac{1}{3}M$) ($J=50-\frac{1}{3}M$)

9. Yes. Joe's kite string was 50 feet longer than half as long as Mike's. That's getting pretty tricky, isn't it? But you should read some contracts, or some wills, or some deductible insurance policies! You can be sure most lawyers have studied algebra!

10. You have just been considering relationships between two or three things, only one of which we have been treating as an unknown, because the relationship of another quantity to it was simple, such as twice as long, or old, or what's remaining from a given quantity. In a later program, we will deal with problems about two or more unknowns, and see equations like $4X+2=3Y-5$ or $X^2+3Y^2-4Z^2=100$. But to find exact values for each of these unknowns, we may need to express several relationships in several different

equations. Can we get an exact value of X from the equation X=2Y if we don't know the value of Y? (Yes) (No)

11. Right, we merely know one relationship between two unknown quantities, which we could express in words, or in an equation like the one given, or on a graph as a straight line. More complex relationships are graphed as curved lines, and are written in equations which include the unknown quantities in squared or cubed terms, or with some exponent other than one.

12. If we know two independent relationships between two unknowns,we can evaluate them; that is,find specific values that satisfy both relationships. But if we have just one equation or formula and two or more unknown quantities, these quantities may vary to any value. These unknown quantities are called variables. In the equation X=2Y, what is X? (variable) (constant) (subtrahend)

13. Yes, it's a variable. And so is Y. But what do you suppose we should call the "2"? (variable) (constant) (subtrahend)

14. Right. The number 2 is just a fixed, constant arithmetic number whose value we've agreed on. If we gave the cashier X nickels and got Y dimes, X would equal 2Y,and if we got back 5 dimes, which is the value of Y, X would be 2 times 5, or 10 nickels.

15. If you gave the cashier a dollar bill and asked for nickels and dimes, and you got 12 coins, how many nickels did you get? As a memory helper this time,let's let the number of nickels equal "N" for being worth five cents, and the number of dimes we'll call "D".

16. We could say that 10 times D plus 5 times N is 100, but we know that in this case D is just 12-N, so without using a complex procedure, we'll merely write 10 times the quantity 12 minus N plus 5N is 100. Now first you need to perform the indicated operation, simplify, collect terms, and so on without changing values for the left side; then perform some identical operation on both sides to change the values and solve the equation. The first indicated operation would give what results? [(12-10N)+5N=100] [120-10N+5N=100]

17. Yes. Next we would collect like terms. -10N and +5N are like terms; algebraically added, the sum is -5N. Now we have 120-5N=100. What do we do next to both sides? (subtract 120, -5N=-20) (multiply by 5, get N=30)

18. Yes. Now you know that minus 5 times the number of nickels is minus 20 cents; or if you simply multiplied both sides of the equation by minus one, you could say that five times the number of nickels you got in change was worth 20 cents. Divide both sides by the 5 cents which a nickel is worth, and you got how many nickels in your dollar's worth of change? (3) (4) (5)

19. Right! We will go through a few simple problems like this, but before we do, here are a few hints about writing equations expressing relationships which are stated in words. When you hear words or phrases such as "more than," "above," "higher," "the sum of," "added to," "and," "as well as," or "the total," what kind of operation would you expect? [+(addition)] [÷(division)] [$\sqrt{\text{square root}}$]

20. Yes. If you are told about "fewer," "the difference," "the remainder," "reduced by," "decreased," "lowered," "dropped," "less than," and so on, what operation would you look for? [X(multiplication)] [-(subtraction)] [X^2(squaring)]

21. And if you notice words like "the product," "multiplied by," "at the rate or price of so much per gallon," or the word "of," as in "half of," or a "third of," or the word "times," what operation is it? (addition)

(multiplication) (subtraction)

22. Division is indicated by phrases like "divided by," or "divided into," "the ratio of," "that part of," or "percent," "fraction," "share," "portion," or the like.

23. Frank and Dave had just enough money between them to buy a used motorbike for \$100. But Dave contributed three times as much money to the purchase as Frank. How much money did Frank contribute? Well let's say Frank put in "F" dollars. How would you express the number of dollars Dave contributed? (F) ($\frac{1}{3}F$) ($3F$)

24. Correct. And Frank's "F" dollars plus Dave's "3F" dollars added up to \$100. How would you write the equation? ($F+3F=100$) ($3F-F=100$) ($3F=133.33$)

25. Yes. And combining the like terms on the left side gives $4F=100$. How much did Frank contribute: "F"? ($F=\$40$) ($F=\$25$) ($F=\$20$)

26. Shirley and Linda pay between them \$130 a month for their apartment, but Shirley owns most of the furniture so she pays \$10 a month less than Linda toward the rent. How much does Shirley pay? Can we say that $S+(S+10)=130$? (Yes) (No)

27. Yes. And Linda's payment of $(S+10)$ can be brought out simply from the parentheses, since no multiplications are needed and no negative signs require changes. Now we have $2S+10=130$. What next? (subtract 10 from both sides, $2S=120$) (multiply by 10 and add $2S$)

28. Right. $2S=120$, so divide both sides by 2 and we get $S=60$; Shirley pays \$60 and Linda \$70 of the \$130 rent. In this problem and the other earlier problems,we worked with only one unknown in the equation, but these were actually two unknowns and two relationships. What we did was to quickly substitute the second of the unknowns, Linda's payment, with another expression. This expression is based on the first unknown quantity of the second simple relationship. Then we solve the first equation. This substitution procedure wouldn't be very easy if both of the relationships were long or complex.

29. If a man travelled 100 miles, first driving X miles in his car to the train station, then riding twice as far on the train, then riding the last 10 miles in a taxi, how far did he drive his car to the station? Express this in an equation with one unknown. ($X+Y+T=100$) ($X+Y+10=100$) ($X+2X+10=100$)

30. Right, and collecting X and $2X$ we have $3X+10=100$. What is the next step? (subtract 10 from both sides, $3X=90$) (divide both sides by 10, $X=40$)

31. Right, and when $3X=90$, we can divide both sides by 3 and find that he drove 30 miles to the station. Perhaps he should have driven the whole 100 miles in his car, but his wife needed it to go to a shopping center in the other direction.

32. The Giants played 50 games and lost 10 more games than they won. How many did they win? Which equation do you use? ($X-10X=50$) ($X+(X+10)=50$)

33. Yes, and since $2X+10=50$, then $2X$is 40, and X is 20 games won. Was it a good season? (Yes) (No)

34. Mrs. Baxter bought 24 square yards of carpet listed at \$10 per yard from a roll three yards wide. How long was her piece of carpet? Which equation would be a good start to the solution? ($3X=24$) [$3(X+10)=24$]

35. Right. Don't bother with information that doesn't affect the answer. She got a piece of carpet 8 yards long and 3 yards wide, or 24 square yards, since area is width times length. Maybe she got a discount from list price. Let's hope so.

36. Mrs. Baxter got another piece of 3-yard wide carpet and asked for binding on all four sides, which required 16 running yards of binding. How many yards long was the piece of carpet? Choose an equation. [3(X-16)=24] [2X+2(3)=16]

37. Yes, you remembered the perimeter, a distance around a rectangle, is twice the width plus twice the length.

38. If you wish to learn a short cut in solving equations, here's one, but be very careful in applying it. It's called transposition, and it's just a quick mental way of solving equations by doing the same addition or subtraction operation to both sides without spelling it out. We just move a term over to the other side of the equation and change its sign!

39. Dick's brother is 6 years older than he is and lacks 4 years being twice as old as Dick, so we can say D+6 is his brother's age; also, 2D-4 is his brother's age, so 2D-4=D+6. What two operations could we do to get the unknown on the left side and the constants on the right side of the equation? Remember we must change the sign of any transfers! (transfer -4 to the right side (as +4), transfer D to the left side (as -D), get 2D-D=6+4) (transfer -4 to the denominator of D+6, get $\frac{D+6}{4}$)

40. Right. And combining terms: 2D-Dis D, 6+4 is 10, and Dick is 10 years old. It may take some practice to do this kind of quick transposition. You may prefer to do it the slow way for a while.

41. Don't forget, you should repeat this lesson several times to be sure you understand and remember all of the rules of algebra you have learned.

Fractions and Division

1. You have already studied examples of fractions in algebra which occur as coefficients. For example: Bill's kite string is only half as long as Tom's, so we say $\frac{1}{2}T=B$, or $\frac{T}{2}=B$, and so on. We could have constant coefficients written like $\frac{2}{3}X=Y$ or $\frac{2X}{3}=Y$, but these fractional coefficients are not exactly what we will call an algebraic fraction.

2. We'll call an algebraic fraction one which has a letter representing an unknown, or a variable, on the bottom side, (the denominator), of a fraction. This is a little harder to deal with, so we'll devote nearly all of this program to such fractions. When a fraction has a letter in the denominator, what is it called? (algebraic fraction) (mixed fraction) (unknown fraction)

3. Yes. Algebraic fractions are like arithmetic fractions, but you must be careful in using them especially when polynomials occur because you must consider either expression above or below the division line as a whole. Consider the numerator or denominator of the fraction as an expression enclosed in parentheses to which a multiplication, (or in this case division), is being applied in its entirety.

4. For example, you know that in dealing with a fraction in order to combine it or simplify it, you sometimes multiply or divide both the numerator and denominator by some factor. This doesn't change the value of the fraction, as you remember. But before we study algebraic fractions, let's review how polynomials are treated in the numerator. If the numerator or the denominator is a binomial, you must remember that any new factor must be applied to both terms, not just the first term. X+1, over 2 is the same as 2 times the quantity in parentheses X+1, over 4. But it would be wrong just to change it to 2X+1 over 4!

5. What fraction is the same as $\frac{Y+3}{5}$? $(\frac{(2Y+3)}{5})$ $(\frac{(2Y+3)}{10})$ $(\frac{(2Y+6)}{10})$

6. Right. You may show a factor outside a polynomial in parentheses or you may multiply the factor by each term. What fraction has the same value as $\frac{3X+5}{4}$? $(\frac{(6X+5)}{8})$ $(\frac{2(3X+5)}{8})$ $(\frac{(23X+10)}{8})$

7. Yes. You may show a factor outside a polynomial in parentheses. What fraction is the same as $\frac{7}{(3X-6)}$? $(\frac{7}{3(X-6)})$ $(\frac{7}{3(X-2)})$

8. Right. It helps in solving algebraic problems to reduce a fraction to its lowest and simplest terms. What would be the simplest way to show $\frac{3X+9}{12Y-6}$? $(\frac{3(X+3)}{(12Y-6)})$ $(\frac{3(X+3)}{3(4Y-2)})$ $(\frac{X+3}{2(2Y-1)})$

9. Right. Let's see if you can remember some factoring methods from an earlier program. How would you reduce the quantity X+Y, squared, over X^2-Y^2 to its lowest terms? $(\frac{X^2}{X-Y})$ $(\frac{X+Y}{X-Y})$ $(\frac{X^2-2XY+Y^2}{X^2})$

10. Yes. (X+Y) "quantity squared" is X+Y times X+Y; and you may recall that X^2-Y^2 can be factored to X+Y times X-Y. This allows us to cancel out X+Y; that is, divide both numerator and denominator by it, which we can do without changing the fraction's value.

11. Remember: first, cancellation is a division operation. When a quantity is divided by itself, the result is 1—not 0. But in most cases, we don't write the 1 as a factor, or a denominator. Second, in a division operation, you must factor each term in any polynomial before cancelling a factor. You cannot just

cancel out a single term in a polynomial fraction. What is $\frac{2X+1}{4}$ in its lowest terms? $(\frac{X+1}{2})$ $(\frac{2X+1}{4})$

12. Yes. You remember from your study of fractions in arithmetic that $\frac{2}{3}$ times $\frac{2}{3}$ is $\frac{4}{9}$. To multiply fractions, you just multiply numerators together and the denominators together. $\frac{1}{2}$ times $\frac{1}{2}$ is $\frac{1}{4}$; $\frac{2}{5}$ times $\frac{1}{3}$ is $\frac{2}{15}$; and so on. What does $\frac{X}{3}$ times $\frac{2}{Y}$ equal? (6XY) $(\frac{X}{6Y})$ $(\frac{2X}{3Y})$

13. Right. And you remember that to divide fractions, you invert the divisor, and then multiply numerators and denominators. To divide $\frac{2}{3}$ by $\frac{5}{7}$, you'd invert the $\frac{5}{7}$, multiply $\frac{2}{3}$ by $\frac{7}{5}$, and get $\frac{14}{15}$. What's the result of dividing $\frac{X}{2}$ by $\frac{Y}{3}$? $(\frac{XY}{6})$ $(\frac{2Y}{3X})$ $(\frac{3X}{2Y})$

14. Yes. What's the result of $\frac{X}{Y}$ divided by $\frac{X}{2}$? $(\frac{2}{Y})$ $(\frac{2Y}{X^2})$ $(\frac{Y}{2})$

15. Right. Now divide $\frac{9A^2}{5B}$ by $\frac{6A}{25B^2}$. Use some paper if you wish. $(\frac{5B}{3A})$ $(\frac{15A}{2B})$

16. Yes. With fractions, the operations of multiplication and division are similar to the division operation indicated in the fraction already. Of course, the factoring step takes some effort, but it is relatively straightforward.

17. The addition and subtraction of fractions, however, require a different and additional step. Addition of number fractions which don't have a common denominator requires these steps: factoring, selecting the lowest common denominator, multiplying both numerators and denominators, adding the numerators, then perhaps reducing the result again.

18. For example, you recall that to add $\frac{1}{3}$ and $\frac{1}{6}$ we determine that the 3 in $\frac{1}{3}$ is a prime number; then that the 6 in $\frac{1}{6}$ can be factored into 2 and 3. We then select 6 as the common denominator, multiply $\frac{1}{3}$ (top and bottom) by 2, to get $\frac{2}{6}$, then add $\frac{2}{6}$ and $\frac{1}{6}$ and get $\frac{3}{6}$; then reduce again by dividing top and bottom by 3 and get $\frac{1}{2}$! You probably are ready, by now, to buy a pocket calculator!

19. What is the sum of $\frac{1}{4}$ and $\frac{2}{3}$? $(\frac{3}{4})$ $(\frac{5}{6})$ $(\frac{11}{12})$

20. Yes. Remember, after finding the least common denominator, multiply both numerator and denominator by the number needed, then add the numerators. What is the sum of $\frac{2}{X}+\frac{3}{2X}$? $(\frac{6}{X})$ $(\frac{7}{2X})$ $(\frac{5}{2X})$

21. What's the sum of $\frac{2X}{3}+\frac{X}{5}$? $(\frac{13X}{15})$ $(\frac{2X^2}{15})$ $(\frac{10X}{15})$

22. Yes. Another way of looking at this problem would be to factor out X from each term, to get X times the quantity $(\frac{2}{3}+\frac{1}{5})$ which is $\frac{13}{15}$. This is possible because then the two terms would be "like terms."

23. The lowest, or as some say, the "least" common denominator, is the smallest-valued algebraic expression into which you can evenly divide the denominators you are to add or subtract. This means that to be sure this denominator is the smallest possible, you must factor the given denominators into their "prime" factors. This is one reason why you were previously given some practice in factoring algebraic expressions.

24. This should be easy. What is $\frac{X+Y}{2}$ minus $\frac{X+Y}{3}$? First, we find the least common denominator. It's simple to see that we can convert each expression to a form with 6 in the denominator. Thus we have $\frac{3(X+Y)}{6}-\frac{2(X+Y)}{6}$ What is the difference? $(\frac{5(X+Y)}{6})$ $(\frac{(X+Y)}{3})$ $(\frac{X+Y}{6})$

25. Right. Now, what is the sum of these fractions? $\frac{3}{X}+\frac{5}{2X}+\frac{7}{3X}$ $(\frac{47}{6X})$ $(\frac{23}{2X})$

26. Yes. Now, here's a tricky one. Be careful. Use some paper if you need to. What's the sum of these fractions? $\frac{1}{X+Y}+\frac{1}{X-Y}$ $(\frac{2X}{X^2-Y^2})$ $(\frac{X^2+2XY+Y^2}{X^2-Y^2})$

27. OK. Now subtract these same fractions. What is the difference? $\frac{1}{X+Y}-\frac{1}{X-Y}$ $\left(\frac{X^2-2XY+Y^2}{X^2-Y^2}\right)$ $\left(\frac{-2Y}{X^2-Y^2}\right)$

28. Very good. Now you might try a few mixed expressions, like $2+\frac{1}{2}$ or $X+\frac{3}{Y}$. An important thing to remember is that any non-fraction expression may be considered as a fraction with "1" as the denominator.

29. How would you add $X+\frac{3}{Y}$? $\left(\frac{X}{1}+\frac{3}{Y};\ \frac{XY}{Y}+\frac{3}{Y};\ \frac{XY+3}{Y}\right)$ $\left(\frac{Y(X+3)}{XY};\ \frac{(X+3)}{Y}\right)$

30. Yes. One of the causes of errors in dealing with algebraic fractions, as with whole numbers, is the handling of signs. One or more of the terms of a polynomial in the numerator or the denominator may be negative. The first term is usually positive, with the plus sign omitted. These conditions may cause you to make mistakes because of the extra steps involved, even though quantities with positive and negative signs are actually treated in the same way as in whole numbers or undivided expressions.

31. As in multiplication, a division expression does not change in value if the signs of both of the multipliers, or of both the numerator and denominator, are changed at the same time. The product of (p) times (q) is the same as (-p) times (-q), and the same is true for p over q and -p over -q.

32. For example, add $\frac{5}{X-Y}+\frac{3}{Y-X}$. This is really the same as $+\frac{5}{X-Y}+\frac{3}{-X+Y}$. What would be a solution? $\left(\frac{5(-X+Y)}{X-Y}+\frac{3}{X+Y};\ \frac{15X}{X-Y}\right)$ $\left(\frac{5}{+X-Y}+\frac{(-3)}{(+X-Y)};\ \frac{2}{X-Y}\right)$

33. Right. Remember, polynomials which act as numerators or denominators of algebraic fractions must be treated as if they have parentheses or brackets around them. You can't operate with either a factor or a sign change on just one term of a polynomial.

34. Let's do a simple review problem. Last year, a 99-pound ancient Greek statuette made of bronze was found in the Mediterranean Sea. Bronze is an alloy of copper with some tin and a little zinc. This statue had eight times as much copper and twice as much tin as zinc. To find how many pounds of zinc was in it, which equation would you use? (8Z+2Z+Z=99) ($8Cu+\frac{Tn}{2}$-Z=99)

35. Yes. Combining the "Z" terms, 8Z+2Z+Z=99, we get 11Z=99, so Z=9 lbs. of zinc. How many pounds of copper was in the statue? (Cu=8Z=72 lbs.) (Cu=2Z=18 lbs.)

36. OK. A telegram sent from Chicago to St. Louis cost $1.00 for the first 15 words and 2 cents per additional word. Using "W" for the number of words, pick an equation which would serve as a formula to compute the cost "C" in cents. (C=100+2W) (C=100+2(W-15))

37. Right! Now let's simplify C=100+2(W-15) by performing the indicated multiplication. We get C=100+2W-30. Combining terms, C=2W+70. If you sent a message of 50 words, what would it cost? (C=2(50)+70; C=170¢=$1.70) (C=250+70; C=320¢=$3.20)

38. Yes. In this case, however, we'd somehow need to say that C could never take a value less than 100, or one dollar, no matter what the number of words. And the number of words would always be positive.

39. Remember that in the multiplication of algebraic terms, the result is called the product. What is a word for the multipliers? (factors) (solutions) (factories)

40. Yes. What may you do to an algebraic fraction without changing its sign? (add -1 to both numerator and denominator) (multiply numerator and denominator by the same quantity)

41. Don't forget, you should repeat this lesson several times to be sure you understand and remember all of the rules of algebra you have learned.

Dorsett Educational Systems, Inc. Box 1226 Norman, Oklahoma 73070 (405) 288-2300

Solving Problems With Equations

1. In the previous program you learned how to deal with algebraic fractions which have letters in their denominators. Earlier, you learned about equations with constant coefficients which were fractions. Now you will learn how to solve equations containing fractions, both with ordinary numbers and with algebraic fractions.

2. We know how to combine $\frac{X}{2}+\frac{X}{3}$ into $\frac{5X}{6}$ or $\frac{5}{6}X$. We factored out X to get $(\frac{1}{2}+\frac{1}{3})$ times X, then added the numerical fractions by the regular arithmetic procedure of finding the least common denominator, and so on. And if we had an equation $\frac{X}{2}+\frac{X}{3}=5$, then $\frac{5X}{6}$ would equal 5. What would X equal? (divide both sides of equation by $\frac{5}{6}$, X=6) (multiply both sides by 5, 6X=25, X=$4\frac{1}{6}$)

3. Yes, of course. If we had an equation such as $\frac{X}{A}+\frac{X}{B}=5$, we could use AB as a common denominator to get $\frac{BX}{AB}+\frac{AX}{AB}=5$. Then we combine terms and factor X to get $(\frac{A+B}{AB})$ times X=5, and solve by dividing by the fraction to get X=5 times $(\frac{AB}{A+B})$. In this solution we treated A and B as letters representing values which would be known quantities in usual situations, such as using a formula. What would be the solution for X, in $\frac{X}{D}+\frac{X}{E}=4$? (X=4DE) (X=4(D+E)) (X=4$(\frac{DE}{D+E})$)

4. Right. Most formulas are letter equations with the letters representing the names of the values in the formula. For example, in the formula A=LW, Area, Length, and Width of a rectangle are represented. Do you remember what the letters stand for in the formula $A=\frac{1}{2}bh$? (Area=$\frac{1}{2}$ base times height (of a triangle)) (Ale=$\frac{1}{2}$ barley times hops (a mixture)) (Angle=$\frac{1}{2}$ base times head (in a circle))

5. Yes. But in many problems, when you don't remember the formula, you can work out the actual relationships of the quantities simply by applying some common sense and watching for the "key" words suggested in Program 7, such as "the sum of," "the remainder," "multiplied by," or "that part of."

6. If you are to find only one quantity, select a letter such as X, Y, or Z, or perhaps an initial of the unknown quantity, if it won't be confusing. If two or three related unknown quantities are involved, it's probably a good idea to select the smallest value as the unknown, so that you may work with larger number coefficients, instead of fractions. Which would be easier to work with? (2X, X+2) ($\frac{1}{2}$X, X-2)

7. Yes. Then write the equation and read it back to yourself to be sure it states the problem correctly. If so, solve it to find the unknown. Then go back, if necessary, and find the other unknowns from their internal relationships. Always check your answers by substituting the variable for the answer. and working through the equation.

8. In the Mp series of mathematics programs you can study decimal and percentage problems which were simple enough to solve without algebra. Let's try a simple percentage problem and use an equation to solve it. Sarah Jones inherited 12% of her aunt's estate and got \$1800. To find the amount of the entire estate, how would you write the equation? (.12(X)=1800) (X=(.12)1800) (X=$\frac{.12}{1800}$)

9. Yes. And solving for X by dividing both sides by the coefficient 12 hundredths, we get 1800 over .12, or \$15,000 for the amount of her aunt's entire estate.

10. Mrs. Jones put \$1000 of her inheritance in a savings and loan company, and the \$800 remainder in a

bank savings account at 4% interest. Her total annual interest was \$92. What was the annual rate paid at the savings and loan? We know that Income equals principal times interest rate times time; in this case one year. So 92=(1000) times (X) times (1)+(800) times (.04) times (1), and combining, 92=1000X+32. What is X? ($X=\frac{92}{1000}$ =9.2%) (92-32=1000X; 60=1000X; X=.06 or 6%)

11. Right. A number of problems on any general math test are about percentage. The statement of the relationships of the time period, principal amount, interest amount earned and interest rate per year can often be best expressed by an algebraic equation. You remember the formula I=prt, of course.

12. The r, which means interest rate in the interest formula, is a rate, or ratio, of two or more things: in that case,money earned per time, per amount invested. A rate or ratio implies a division, or fractional relationship. Often a decimal fraction has an implied formal denominator, as 100 in percentage. A rate could be miles driven per gallon consumed by an automobile, or average number of miles driven per hour on a long trip. If you drove 240 miles in 4 hours, what would you know? (average rate was 60 mph) (achieved 60% of trip average)

13. Yes. How fast would you have to drive while returning the 240 miles in 4 hours elapsed time, if you stopped for an hour during the trip for lunch? Rate = miles divided by hours, or miles per hour; so R=240 miles divided by the quantity, 4 hours minus one hour. How fast is that? (70 mph) (80 mph) (90 mph)

14. Correct. Fred Brown left home at one o'clock one afternoon, drove at an average of 40 miles per hour to Princeton, where he had a two-hour business meeting, then drove home at an average of 50 miles per hour in time to see the six-o'clock news. How far does he live from Princeton? We'll try to set up an equation to find out.

15. We know that the rate (R) is distance over time. We can solve for distance, by multiplying by time, to get "distance equals rate, (R), times time, (T)." D is the unknown we wish to find. The two rates, we already know, but we don't know the time required for either trip. We can find out the times, however, since Mr. Brown was gone for 5 hours, from 1 'til 6 o'clock, and he spect 2 of those 5 hours at the metting. What does this tell us? (total time for both trips was 3 hours) (the difference between 1 to 6 p.m. is the same as 5 hours less 2)

16. Yes. Now we could write two equations, one for each trip, but since the desired unknown, the distance D, is the same for both equations, we'll just write one. The distance is the rate, which was 50 miles per hour going home, times the time required to go home. This can be expressed as 3 hours for the round trip, less the time used going to Princeton. How can we write, in an algebraic expression, the time he used going to Princeton at 40 miles per hour? (Time=distance-rate; $T=\frac{D}{40}$) (Time=DR; T=40D)

17. Correct. We always solve the relationship for the quantity we need. Now we know that the distance D he drove home is equal to 50 miles per hour times the time required. This in turn is 3 hours less the time required to go D miles at 40 miles per hour. What equation do we finally use? [D=50(3-40D)] [$D=50(3-\frac{D}{40})$]

18. Right. Now let's solve this simple equation. First we'll perform the indicated operations on each side. Since there are no like terms in the parentheses, we will next do multiplication by 50 and get $D=150-(\frac{50D}{40})$; or with a little cancellation, $D=150-(\frac{5}{4}D)$. What do we do next to solve for D? (subtract D from both sides) (add $\frac{5}{4}D$ to both sides)

19. Right. Then we get $D+(\frac{5}{4}D)=150$. Of course you can see by now that to isolate D, the unknown, on the left side, we'll have to divide both sides by $\frac{9}{4}$, which is the same as multiplying both sides by $\frac{4}{9}$. What do we finally get as the distance home? [$D=(\frac{4}{9})(150)$=66.6 miles] [$D=(\frac{5}{9})(150)$=83.3 miles]

20. Yes. This was a relatively complex problem which was easily solved by the logic of algebra. Incidentally, it

would have been tempting to multiply the average time for the trips, $1\frac{1}{2}$ hours, times the average speed, 45 miles per hour, and get $67\frac{1}{2}$ miles; not far from correct, but not right. And if the speeds were greatly different, going and coming, the error would be much greater.

21. Fred and his family were moving to St. Louis. His wife, Eleanor, started early one day driving the station wagon at 50 miles per hour. Fred paid some bills and started 2 hours later in the sedan, which he drove at 70 miles per hour. How far had they driven before he caught up with his wife? This is similar to the last problem using $R=\frac{D}{T}$ and D=RxT. The two distances are the same, the two rates are known, but instead of knowing the sum of the travel times, as in the round-trip problem, we know their difference: the two-hour late start.

22. Let's set the distance, "D," equal to the 50 miles per hour rate at which Eleanor drove, times the time she required to drive D miles. But we'll also say D is equal to her speed of 50 miles per hour times the quantity in parentheses: 2 hours plus Fred's driving time. Fred's driving time was the distance D, divided by his speed, 70. So what equation can we use to find D? ($D=\frac{50}{70}+2D$) ($D=50(2+\frac{D}{70})$)

23. Right. Now let's solve the equation by multiplying each term in parentheses by 50; we get D=100 plus $\frac{5}{7}$D. What do we do to both sides to get all the unknown terms on the left side? (subtract $\frac{5}{7}$D) (add D) (multiply $\frac{7}{5}$D)

24. Yes. $\frac{2}{7}$D equals 100. Next, of course, we get rid of the $\frac{2}{7}$ coefficient of our unknown, D, by dividing both sides of the equation by $\frac{2}{7}$, which, as we know, is the same as multiplying by $\frac{7}{2}$. How far had Fred and Eleanor driven before he caught up with her? (250 miles) (350 miles) (450 miles)

25. Correct. It really doesn't make much difference if the rate, or ratio, is miles per hour, miles per gallon, gallons per hour, or assemblies per shift. If you have a rate, R, which is so many A's per B, then the average rate is $R=\frac{A}{B}$. And A=R times B and $B=\frac{A}{R}$, as you solve for each of the other quantities.

26. We have just solved a rate problem with two given rates, a common unknown distance to be found, and a given difference between two unknown times. Now let's try one with a common and given time, two unknown distances whose total is given, and two unknown rates whose difference is given.

27. Mike called his girl friend 500 miles away and they both agreed to start out at once to meet each other in Phoenix. "By the time I get to Phoenix she'll be arriving," Mike said, and drove 10 miles per hour faster than his girl to get there when she did. They met in 4 hours. Which equation will tell how fast he drove? ($R+(R-10)=\frac{500}{4}$) ($R=\frac{4(R-10)}{500}$)

28. Yes. You might say that the total distance was covered at the "closing rate," the sum of these two speeds, which, of course, was R+(R-10), where Mike's rate is R. Let's solve for R. Since there's no multiplication indicated to the binomial in parentheses representing his girl's speed, we'll just drop the parentheses, add the R's and get 2R-10=125 miles per hour. Then what is R? (subtract 10, multiply by 2, R=115 miles per hour) (add 10 to both sides, divide both sides by 2, R=67.5 miles per hour)

29. Right. Another use for algebra is to deal with relationships among the quantities in space, as described by geometry series, to get familiar with the regular, ordinary, or standard relationships in common geometrical shapes. You recall that an equation is an algebraic statement of equality showing the relationship between quantities. What is a formula? (an equation showing standard relationships) (a secret, showing how to blow it up!)

30. Yes. You should remember the formula for the area of a rectangle. What is it? ($A=LW$) ($A=\pi W^2$)

31. Yes. Now solve $A=LW$ for the length, L. ($L=\frac{A}{W}$) ($L=AW$) ($L=\frac{W}{A}$)

32. Yes. A "hectare" is metric area equal to about $2\frac{1}{2}$ acres, since a square hectare is 100 meters square, containing 100 times 100, or 10,000 square meters. If you bought 2 "hectares" of land in Mexico in a rectangle that was only 50 meters wide, how long was it? ($L=\frac{A}{W}=\frac{20,000}{50}=400$ M.) ($L=AW=\frac{(100)^2(50)}{10,000}=500$ M.)

33. Yes. When her sister came over for coffee, Norma had a fourth of a pie left in the refrigerator. She cut it into two pieces, and gave her sister a piece that was 10 degrees bigger than her own. How big was Norma's piece? ($R=\frac{90^\circ}{2}-10$; $R=35^\circ$ piece of pie) ($R+(R+10)=90^\circ$; $2R=80^\circ$; $R=40^\circ$ piece)

34. Right. The supermarket sold hamburger at 88 cents a pound and some better ground sirloin at $1.08 per pound. They wanted to run a weekend special on a mixture of the two, but with the same profit on each grade of meat, and call it "special ground beef, 97cents." How many pounds of hamburger did they use for each 100 pounds of the special mixture? Well, we know that 88 cents times H, the amount of hamburger, plus $1.08 times (100-H) is going to bring 97 dollars, don't we? (yes) (maybe) (probably not)

35. Yes. So we write $.88H+1.08(100-H)=97$. Now do the indicated multiplication and you get $.88H+108-1.08=97$. What two steps do we do next? (add 97 to both sides, divide both sides by .88) (add +.88H to -1.08H, (left side), subtract 108 from both sides)

36. Right. When you do this you get $-.2H=-11$. What next? (divide both sides by -.2; H=55 lbs.) (subtract -.2 from both sides; H=11.2 lbs.)

37. Yes. Apparently the weekend special was mostly hamburger, whatever they called it. The bulk wine bottler trying to hold the alcohol content to a certain specification is having the same kind of problem. Sometimes he needs to remove alcohol, but this year his wine has only 5% alcohol, and he needs 6%. He has 5000 liters of wine; how many liters of alcohol does he need to add? Let's see.

38. The wine now has .05 times 5000 liters, that is,250 liters, of alcohol; then we add X liters of alcohol. It then totals, when added to the original 250 liters, 6% of the resulting (5000+X) liters of wine. How do we say that in algebra? [$X+250=.06(5000+X)$] [$.05(X+250)=.06(X+5000)$]

39. Yes. Now let's solve the equation. Perform the indicated multiplication on the right side of the equation. This gives $X+250=300+.06X$. Now, to get a single X on the left side and a single number on the right, what do we do? (subtract .06X, divide by 250, X=120 liters) (to both sides, subtract .06X; subtract 250, divide by .94; X=53.2 liters)

40. Right. We took those three steps rather quickly, but with a paper and pencil you should be able to follow them without trouble. In addition to the kind of mixture, rate, percentage, and geometry problems which are aided by algebraic statements showing relationships, there are lots of common types of problems,such as age problems, denominational problems, including coinage and packaging quantities, and the like. Later we will study relationships which are even more complex, but still can be dealt with by the logic of algebra.

41. Don't forget, you should repeat this lesson several times to be sure you understand and remember all of the rules of algebra you have learned.

BASIC ALGEBRA

Ratio, Proportion and Variation

1. We often can deal with things by comparing them to something else we know about. When we say something is bigger or taller or older or faster, we may refer to the difference between things. A difference is obtained, you remember, by subtraction. When we say something is half as expensive or 8 tenths as large, we are implicitly referring to what operation? (addition) (multiplication) (division)

2. Right. And when we make a comparison by division, we are using a ratio. A ratio is a fraction, with the thing we are considering usually as the numerator, and the thing we are using as a reference, or comparison, as the denominator.

3. Like a fraction, a ratio should be reduced to lowest terms for easiest handling. The ratio of Bobby's height, 40 inches, to Jim's, 60 inches, is 2 to 3, or $\frac{2}{3}$. When you mentioned some value first, in what position does it go? (numerator) (middle) (denominator)

4. How would you show, as a fraction, the ratio of Pete's age, 24, to his sister's age, 16? ($\frac{42}{16}$) ($\frac{2}{3}$) ($\frac{3}{2}$)

5. Right. But what is the ratio of 3 pints to 2 quarts? ($\frac{3}{2}$) ($\frac{2}{3}$) ($\frac{3}{4}$)

6. Right again. We must convert the values in the ratio to the same units; otherwise, it is not a ratio. What is the ratio of 8 ounces to 2 pounds? ($\frac{1}{4}$) ($\frac{8}{2}$) (4)

7. Yes. Sometimes we do a division to find a rate. For instance, you drove 100 miles in 2 hours, so your average rate of speed was 50 miles per hour. But a rate is different from a ratio, for a ratio results from the division of two quantities of exactly the same kind. What is a ratio? (rate of speed) (comparison by division) (comparison by subtraction)

8. Yes. And don't forget, the fraction set up by the division operation should be reduced to lowest terms, and in other ways treated as a fraction. If your car is 18 feet long and 6 feet wide, what is its ratio of length to width? ($\frac{1}{3}$) ($\frac{18}{6}$) (3 to 1; 3; or 3:1)

9. Right. A common ratio is the "aspect ratio" of a picture, movie or television screen. The usual ratio is a width-to-height of 4 to 3. A so-called "15-inch" TV screen might have a width of 12 inches and a height of 9 inches. But the same ratio occurs with "20-inch" TV screen having a ratio of 16 to 12 inches. Are these ratios equal? (Yes) (No)

10. You might say that they are in proportion. In mathematics, a proportion is an equation, or statement, that two ratios are equal. The 15-inch TV screen shows the same picture as the 20-inch screen since their ratios are the same. We could state this as a proportion. We could say that 12 is to 9 as 16 is to 12. How would you write this? (12:9=16:12) (12x9=16x12)

11. Yes. And we could write $\frac{12}{9}$ equals $\frac{16}{12}$. Here is a memory trick. In a proportion written 12 colon 9

equals 16 colon 12, the four numbers can be divided into two groups: the inside numbers and the outside numbers. Math teachers have called these the "means" and "extremes." The two inside numbers, multiplied together, equal the product of the outside numbers.

12. This is merely saying that when two fractions are equal, the product of the numerator of one fraction times the denominator of the other equals the product of the other two numbers cross-multiplied. This should be obvious by inspection, because if the fractions are equal, they are identical, except for a common factor in both numerator and denominator in one of the fractions. This factor, of course, will be included in each cross-product. What is the product of the means in 8 is to 10 as 4 is to 5? (20) (32) (40)

13. Right. What is the product of the extremes in 8 is to 12 as 4 is to X? (32) (96) (8X)

14. Right, and since the product of the "inside" numbers, or means, is 48, what is X? (4) (6) (8)

15. If $\frac{144}{12}=\frac{X}{8}$, what is X, and what are the ratios? (X=96; ratios are 12:1) (X=84; ratios are 12:1) (X=72; ratios are 9:1)

16. Yes. The Cinemascope screen is 84 feet wide and 36 feet high in the Radio City Music Hall. If your TV screen which is 15 inches high were in proportion, how wide would it be, and how would its diagonal rating be given? Try "means and extremes." (22 in. wide, "25-in. picture") (27 in. wide, "32-in. picture") (35 in. wide, "39-in. picture")

17. That would certainly be a wide television tube, wouldn't it? In geometry and trigonometry you will frequently be dealing with diagonals of rectangles or triangles; and proportion is an important concept. When triangles are in proportion, and the ratios of the three corresponding side lengths are the same, they are called similar triangles.

18. In similar triangles, what do we know about lengths of the corresponding sides? (they are identical) (they are long) (they have the same ratio: $\frac{A}{X}=\frac{B}{Y}=\frac{C}{Z}$)

19. Right. A proportion expressed with more than one equal sign is called an extended ratio. The angles in a triangle or other polygon are considered ratios, too, since they are not true dimensions. What do you think can be said about the corresponding angles in similar triangles? (they're usually similar) (they're always exactly equal) (two of them are equal)

20. Yes. And since similar triangles and parallelograms are always in proportion, we can use our means and extremes method. If you paced off the shadow of the Washington Monument and measured 100 feet, then found that a 50-inch post cast a 9-inch shadow, how tall is the Monument? (means 100x50=extremes Mx9; M=555 ft.) (monument = 528 ft. tall) (extremely $\frac{100}{50}$; means $\frac{M}{9}$; M = 500 ft.)

21. Yes. The lines were formed by the vertical centers of the objects; that is, the shadows on the ground and the sloping shadow lines form similar triangles. Here are two examples of sets of similar triangles. Their apex angles are the same, because they are included, or are made by, line extensions, and the bases are constructed to be parallel. When you know all about one triangle and a little about the other similar triangle, you can calculate the rest.

22. Here is a triangular ladder with horizontal steps. You can see that the width of the steps varies directly with the distance down from the top. This is because the included triangles formed by the top angle and each step are similar, and the sides are all proportional. Things which vary directly are proportional, like the distances traveled at various times when traveling at a constant speed.

23. On the gasoline pump, the total cost of the gasoline delivered to your tank does what, with gallons delivered? (varies directly) (doesn't vary) (varies inversely)

24. Yes, this is true, because there is a fixed ratio, the price per gallon, while the gasoline is being delivered. This is called direct variation.

25. If X divided by Y is constant, then as X becomes larger, Y does, also, in proportion. It varies directly. But if X times Y is a constant, then as X increases, Y decreases. Sometimes X or Y is a rate, like miles per hour. Then if you have a fixed amount, like the distance to your job, the time required to drive it varies inversely with the average speed you drive. And how does the speed vary, over a fixed distance, with the time spent driving to work? (varies directly) (doesn't vary) (varies inversely)

26. Right. Here's the graph of $\frac{X}{Y}=4$ and the graph of $XY=4$. Notice that the equation with a ratio equaling a constant is linear, or a straight line, while the equation with a product of two variables equaling a constant makes a curve. This curve is called a hyperbola.

27. Another way of expressing a linear equation with a constant ratio of two variables is, of course, that one variable equals the constant times the other; then X equals 2Y. Another way of expressing $XY=2$ is to say $X=\frac{2}{Y}$. In this case, how does X vary with Y? (directly) (constantly) (inversely)

28. You remember that when we choose to vary some variable, such as our speed, then the travel time to work depends on it. We call this second variable a dependent variable. The first variable, that is, the variable we decide to change about, has a different name. What do you think it is? (moving variable) (independent variable) (fixed variable)

29. Yes. You will recall that another name for a dependent variable is function. We say that when $Y=2X+3$, Y is the explicit function of X, which we had selected as our independent variable. If we had chosen to vary Y as an independent variable, we might say that X is an implied function of Y.

30. Let's say that the value of Y depends on X in the equation $Y=5X^2$. How does it change as X is increased from the value 1? (decreases slowly) (increases on a straight line) (increases on a steep curve)

31. Right. If your mother is 22 years older than you are, is this relationship expressed as a ratio? (Yes) (No)

32. A ratio is the result of a division operation. What is this like? (a fraction) (a product) (a sum)

33. Yes. Like a fraction, a ratio is best handled by what simplification? (omit the denominator) (reduce to lowest terms) (invert and multiply)

34. What is a short definition of a mathematical proportion? (compared the inverse ratios) (product of proper portions) (statement that ratios are equal)

35. Right. Now select the best definition of a mathematical ratio. (comparison by division) (contrasting rates) (difference of two values)

36. P is to Q as R is to S. Which are the "means" in this proportion? (P and Q) (Q and R) (R and S)

37. If P is to Q as R is to S, what value does P have, in terms of Q, R, and S? ($P=\frac{QxR}{S}$) (P=QxRxS) ($P=\frac{S}{QR}$)

38. At three-o'clock one day, the shadow of the Empire State Building was 1000 feet long, and the shadow of a five-foot boy was 4 feet long. How tall is the building? (800 feet) (1000 feet) (1250 feet)

39. These triangles have a special mathematical relationship. What is this relationship called? (similar) (supplementary) (complimentary)

40. How does Y vary as a function of X in the equation 3XY=17? (directly) (indirectly) (inversely)

BASIC ALGEBRA

Problems With Two Unknowns

1. In previous programs you have seen equations with two or three letters representing quantities. Some of these were standard formulas representing common relationships. When we used these equations to get number answers, however, you will recall that we were able to replace all of the letters with numbers, except one. This was our unknown quantity; X, or Y, or R, or D, and so on. While you will always solve for one unknown at a time, it is not always best or easiest to set up just one equation with one unknown. In this program we will deal with two unknowns.

2. Suppose we said that Ms. X and Ms. Y divided 10 pounds of sugar between them. We could say X+Y=10, but until something else is said, we still don't know the value of X or Y; we know just one relationship. Both values are unknown. Of course, if we also said that Ms. X got four times as much of the sugar as Ms. Y, that is, X=4Y, you could practically do the algebra in your head. What is X? (X=8) (X=5) (X=2)

3. Yes. And Y is 2. We had two unknowns and two different relationships. But how did you know the answer? In earlier programs when you had simple relationships like this, you in effect substituted the value of one unknown, as expressed in one equation, in place of the same unknown in the other equation.

4. We said X+Y=10, and then we said X=4Y. Then why not write "4Y" in the first equation in place of X and get 4Y+Y=10. We'd know that 5Y was 10, Y was 2, and X was 8. This substitution process is really the best way to solve a lot of problems by expressing them in just one unknown quantity. What do we call this process? (synchronization) (substitution) (solution)

5. Yes. There are other methods of solving for two unknowns when there are two independent equations representing two relationships. They include graphical solutions, and taking the difference between two equations, to eliminate one unknown. This last method is called solving simultaneous equations.

6. You have already had some practice solving equations by substituting the value of one unknown as expressed in one equation into the other equation. Later we will solve some simultaneous equations. That is, we'll eliminate one unknown by subtraction. First, let's see how we solve problems graphically. Here's a graph of the equation X=2Y. What's the value of Y when X is 0? (0) (1) (2)

7. Correct, even if X is twice as large as Y, when X is 0, Y is just 0. Notice here we have horizontal and vertical lines crossing each other. The point of intersection is 0 for both the X and Y lines or axes. The X-axis to the left of 0 is negative, and to the right is positive. Above 0 on the Y-axis is positive, and below is negative. If X increased, which way do you think it would move? (upward) (downward) (to the right)

8. Right. And as the value of Y increases, it moves upward on the graph. X and Y are letters which stand for unknown values which can vary, and are therefore called variables. Constants, on the other hand, are numbers or letters which represent known numbers.

9. In most relationships between two variables, we usually assume that a second variable changes as a result of making a change in the first variable. The variable which changes as a result of considering different values of the first variable is called the dependent variable, since it's value varies depending upon what value we are considering in the first variable. Since we use the word "dependent" for the variable, say Y, which varies when we select different values of X, what would you think we'd call X? (arbitrary unknown) (independent variable)

10. Right. We usually choose the letter X as the independent variable. You remember its values change left and right. The dependent variable, Y, moves up and down. This means that generally we think in terms of how much vertical change there is, when we decide to change our horizontal value.

11. Here's another term, "function." In algebra, the dependent variable is said to be a "function" of the independent variable. Let's say you use a gallon of gasoline for each 20 miles you drive. If you take a longer trip, you use more gasoline. $M=20G$, or $G=\frac{1}{20}M$. What variable would be considered a "function" of M, miles driven? (X) ($\frac{1}{20}$) (G)

12. Here is the graph of the equation $X+Y=10$. If Ms. X took more sugar from the 10-pound bag, that left less for Ms. Y. You can see that if either value is zero, the other is 10. You also can see that the graph of the equation is a straight line, just as we previously saw $X=4Y$ was a straight line, but not in the same direction. What do you think we call equations which graph as a straight line? (curvy) (linear) (circular)

13. That's correct. And you can usually identify a linear equation, (graphed as a straight line), because it has only "first order" terms of the unknowns. This means there is only a "one" as the unwritten but understood exponent of each of the unknowns. These will be terms like $2X$, $\frac{3X}{5}$, or $14Y$, etc., not terms like X^2, Y^3, or πR^2.

14. Here are some graphs of linear equations. What do you notice about these graphs? (they're all curved) (they slope down to the right) (they're straight; slope up to the right)

15. Right. And here are some more graphs. Notice that they all slope down to the right. This is because when solved for Y, X is a negative term. The downward slope is called a negative slope. Study this for a moment and then push the center button to proceed.

16. We say that conventionally Y, being moved up and down, is the dependent variable, depending upon the value we assign to X. If you solved $X=2Y+4$ for Y, what would you have? ($Y=\frac{X-4}{2}$) ($Y=\frac{X-2}{2}$) ($Y=\frac{1}{2}X-4$)

17. You've seen the lines on the graphs, but haven't learned how they represent the equations. Let's plot a few. Let $X=Y$. As the value of X increases by moving to the right, the value of Y increases by moving upward. If you begin at the center of the graph with $X=0$ and $Y=0$, you will move on an upward slope through $X=1$ and $Y=1$ to $X=2$, $Y=2$, and so on.

18. With $X=2Y$, if X is 0, Y is 0; if X is 2, Y is 1; if X is 4, Y is 2 and so on. What would Y be if X is 6, and would it be graphed on a straight line through the other points? ($X=6$, $Y=3$; Yes) ($X=6$, $Y=4.735$; No)

19. Yes. And if $X=Y+2$, we find that if X is 2, Y is 0; $X=3$, $Y=1$; X at 4 makes $Y=2$; and $X=5$ gives $Y=3$. It's a straight line, but doesn't go through the 0-0 corner point of the graph, called the origin.

20. As you can see, a point on a graph is read in alphabetical order first S, then Y, so a point P, showing $X=3$ and $Y=4$, would be shown in parentheses as 3, comma 4. How would you refer to a point where X was 2, and Y was 5? (5,2) (2,5)

21. Right. The numbers of the point are called its "coordinates." The coordinates of a point where X is 2 and Y is 3 are 2, comma 3 within parentheses. The vertical coordinate, usually Y, is called the ordinate. But the left-right, or X-value, is called the abscissa.

22. Study this graph carefully and decide which equation is shown. ($2X=Y+3$) ($X=2Y+3$) ($2X=3-Y$)

23. Right. We know that equations represent statements about the relationships of some quantities. When there are two unknown quantities, we need two relationships or equations to find any specific values. In a graph these two equations may be plotted and may cross each other. Where they intersect, the points have values which are true for both relationships, and, in effect, provide a solution that is true simultaneously for both equations. Such graph-plotting procedure is one good way to find a solution, although not always the quickest way.

24. Here is a graph showing equations $X=Y$ and $X+Y=6$. What are the coordinates of the intersection; that is, the solution? (3,3) (3,6) (2,4)

25. Yes, although you probably could have guessed that if X was equal to Y, and their sum was equal to 6, that both X and Y were 3. But how about the solution of $X=2Y$ and $X+2Y=8$? (2,4) (3,3) (4,2)

26. Right. So far we've not gone through the steps required for plotting equations, but you can see that only two points are needed to plot a straight line, and they should be far enough apart for accuracy. Unless X is just some multiple or fraction of Y, one method of selecting two easy points on the graph is to let X equal 0 and evaluate Y, and then let Y equal 0 and evaluate X.

27. In most solutions of two unknowns and two linear equations, we use the procedure of algebraic solution. It is hardly expedient to always plot graphs and find their intersection, but since many persons understand abstract algebraic relationships better when they can be represented by a picture, the procedure of graphing is always helpful to know. Here is the graph of $2X=3Y$ and $2X+Y=8$. What is the solution? (0,0) (3,2) (2,4)

28. Yes. An algebraic solution eliminates one of the unknown quantities represented by letters. You remember we did this earlier by substitution when we could represent one unknown by its equivalent in terms of the other unknown. Besides this substitution method which we have used already, there is the more generally useful subtraction method, which is another way of describing algebraic addition of identical quantities with opposite signs.

29. If you had a pair of equations $X=Y$ and $X+Y=6$, you could change the first equation to $X-Y=0$ by subtracting Y from both sides. Now write one equation above the other. You can see that you can add equals to equals, or subtract equals from equals, and the result will still be an equality. We could add $X-Y=0$ and $X+Y=6$ and the result would have no Y term. We could subtract these equations, and eliminate

the X term. First let's try addition; what do we get? (2X=6; X=3) (2X+Y=6; X=2) (X=6+0; X=6)

30. Yes. And after finding the value of one unknown, we can substitute in either equation to find the value of the other unknown. How would you combine 2X+Y=9 and X+Y=6 to solve for X? (subtract; get X=3) (add; get 3X+2Y=12; X=4)

31. Yes. But what if we have these two equations, 3X=2Y+7 and 2X+Y=0. First, let's get all of the unknown terms on the left sides of the equations and the plain number terms on the right. (Later you will find the so-called "standard" form of some equations calls for a zero on the right side.) In this case, what do we do to place all unknowns on the left and constants on the right? (add 2X to equation II; get 4X+4=0) (subtract 2Y from equation I; get 3X-2Y=7)

32. Right. But we still don't have two equations we can add or subtract to eliminate one unknown. We can, however, multiply both sides of one or the other equation, so that the coefficient of one of the unknowns is the same in both equations. How about multiplying both sides of the second equation by 2. Which unknown would this help eliminate? (X) (Y) (Z)

33. Yes. Now we have 3X-2Y=7 and 4X+2Y=0. What next? (add equations: 7X=7; X=1) (subtract equations: X=7)

34. Correct. Now let's try another one. Take 6X=Y-1 and 5X+Y=12. What do we do first? (put unknowns on left side; subtract "Y" in equation II) (divide both equations by $\frac{5}{6}$)

35. Yes. Now we have 6X-Y=-1 and 5X+Y=12. We add algebraically, eliminate Y's, and find that X is what? (1) (12) (3)

36. Yes. Now for a little harder one. 6X+3Y=4 and 10X-6Y=3 What now? (multiply equation I by 2, add equations) (add algebraically, get 16X=16; X=1)

37. Yes. Now 12X+6Y=8 and 10X-6Y=3. Added, this will give us 22X=11 and X=what? (2) (1) ($\frac{1}{2}$)

38. Right. We have solved two equations with two unknowns by three different methods: 1) by substitution; that is, by substituting one unknown as a function of, or in terms of, the other, when it's simple to do; 2) by graphing; which provides a visible, understandable picture, and shows the solution at the intersection or crossing point of the equation lines; and 3) by subtraction, or algebraic addition, which is called an algebraic solution.

39. When we have two unknowns, and two equations which can solve them, what are they called? (congruative equations) (coexisting equations) (simultaneous equations)

40. Right. An equation which has only first-degree terms of the unknowns, that is, no exponents or radicals shown, will be graphed as a straight line. What is this kind of equation called? (straight-arrow) (linear) (graphical)

41. Don't forget, you should repeat this lesson several times to be sure you understand and remember all of the rules of algebra you have learned.

BASIC ALGEBRA Reference Folder Ma 12

Simultaneous Equations

1. If we said the sum of two unknown quantities was 6, or the difference between two numbers was 2, we could express these relations by writing $A+B=6$, or $C-D=2$. This algebraic notation could be considered a kind of shorthand system. If we said X is 4 units older, or longer, or taller, or in some way bigger than Y, we could jot down $X=Y+4$.

2. If we wished to say the product of the two unknown quantities was 24, we'd write $XY=24$; if one quantity divided by the other was equal to 7, we might say $\frac{X}{Y}=7$, or even $X=7Y$, which says that X is seven times as large as Y and is another way to express the same division relationship.

3. You can show the combination of the addition-subtraction operations with multiplication-division operations. For example, 5 times one unknown quantity plus 3 times another unknown equals 28. Now, how do you think you would say: an unknown quantity is added to 3, and when their sum is divided by another quantity, the result is 15? ($\frac{X}{Y}+3=15$) ($\frac{X+3}{Y}=15$)

4. Right. You can understand that it is simpler to express these relationships in algebraic form than to say them with real precision in ordinary language. As the relationships become more complex, it is helpful to be able to express them in some sort of shorthand.

5. There are related systems of algebraic shorthand to express relationships. Instead of equations, which express equalities, we may deal with inequalities, showing that one expression is greater or less than another. You have already heard about Boolean algebra and set theory which deal with logical relationships.

6. The Boeing 717 seats twice as many coach passengers as first class, and coach passengers are charged only two-thirds as much for their fares as first class. What's the coach fare from Chicago to St. Louis if total fares for a full load of 72 passengers amount to $3,360? Counting fares from each class, how many unknowns are there? (2) (3) (4)

7. Right. Although we are asked to find only the coach fare, let's do the whole thing. Let C equal the number of coach seats, and F the number of first class seats. We know that C is equal to twice F, don't we? $C=2F$. Now let's let X be the coach fare we are asked to find, and Y the first-class fare. Is X equal to $\frac{2}{3}Y$? (Yes) (No)

8. Yes. We also know that C plus F equals 72 passenger seats. Next, the coach fare, X, times coach seats, C, plus first class fare, Y, times first class seats, F, totals $3,360. Is that right? (absolutely) (not quite)

9. Yes. Now we have four equations. Let's do equations 1 and 3. We could solve them either by substitution or subtraction. By solving equation 1 for F, F equals $\frac{1}{2}C$; and we can replace the F in equation 3 by $\frac{1}{2}C$. What would we get then? ($C+2C=72$) ($C=72$) ($C+\frac{1}{2}C=72$)

10. Right. And if $1\frac{1}{2}C$ equals 72, we divide both sides by $1\frac{1}{2}$ to solve for C. This is the same as multiplying

both sides by $\frac{2}{3}$ and we get C, the number of coach seats. What is C? (C=24) (C=48) (C=60)

11. Yes. There are 48 coach seats and 24 first class seats. Now look at equations 2 and 4. Solving equation 2 for Y, we get Y equals $\frac{3}{2}$X, which we can substitute for Y in equation 4. Equation 4, with 48 substituted for C, and 24 for F, and $\frac{3}{2}$X for Y, now reads what? ($48X+24(\frac{3}{2}X)=3360$) ($48X+24X=3360$)

12. Yes. Now perform the indicated operations, which are to multiply 24 times $\frac{3}{2}$ to obtain 36, then add 36X to 48X to get 84X. This gives us 84X=3360. What is the coach fare? ($30) ($40) ($50)

13. Correct. But perhaps you'd better check with American Airlines or TWA before you go. Here's an investment problem. You had $10,000 you inherited from Aunt Martha, and you invested some of it in the Trustworthy Savings and Loan at 6% and the rest in the Swinging Mutual Fund which, it turns out, paid only 4% in dividends. At tax time you forget how much income you got from each, but you remember your check from Trustworthy was exactly $100 more than from Swinging. What do you pay taxes on? Let's see.

14. Let S be your investment in Swinging, and T, in Trustworthy. Then .04S is your dividend from Swinging Mutual Fund, and .06T, your interest income from Trustworthy. What are the relationships we know about S and T? (T+.04T=S+.06S; T+S=5000)(T+S=10,000; .06T-.04S=100)

15. Right. Now let's try to solve these simultaneous equations algebraically; that is, eliminate one unknown by subtraction. This can be done by multiplying the bottom equation by 25, which makes it 1.5T-S=2500. Then you can algebraically add it to the top equation. What's the result? (1.5T=10,000) (T+1.5T=12,500)

16. Right. If T+1.5T, that is 2.5T, is equal to 12,500, T must be $5000. S is also $5000, and you pay taxes on how much dividend and interest income from these two investments? ($10,000) ($5000) ($500)

17. Right, $500. You'd have gotten $600 if you'd put it all in Trustworthy, but maybe next year Swinging will come back up. Or maybe not, because the president of Swinging Mutual Fund wants the company to buy an airplane for him to fly around in, maybe even two airplanes. The company treasurer is horrified. He says they could get four limousines for the Board of Directors for the price of just one airplane. And two airplanes would cost $20,000 more than the entire proposed budget which would get limousines for each of the six directors. What do the airplanes and limousines cost? Let's see.

18. It's easy to see that A=4L. This means that one airplane costs four times as much as each limousine. What do two airplanes cost, according to the treasurer's other complaint? (2A=20,000+6L) (2A=6L-20,000)

19. Yes. Before going any further, let's simplify this second equation by dividing all terms by 2. Now we get A=10,000+3L. Since the first equation was A=4L, when we subtract one equation from the other, we get the limousine cost, L, to be what? (L=$5000) (L=$7000) (L=$10,000; A=$40,000)

20. Right. But perhaps by now you've already cashed in your mutual fund shares, so it doesn't matter. If you and your brother bought a store building for $12,000, but you paid $7000 of the cost and your brother only $5000, how would you divide the year's net profit of $1500, if it's done proportionally to investment? You probably could solve this by a simple arithmetic procedure, but for practice, let's set up the equations.

21. First, your part of the profit plus your brother's is \$1500, so we'll say U+B=1500. Then we'll say that the ratio of your share to the total profit equals the investment ratio, so U divided by 1500 equals \$700 divided by \$12,000. What do we do first? (ignore equation 1; solve equation 2; U=875) (subtract B from both equations)

22. Yes. This is a proportion problem, and since three of the four elements of the proportion are available already, you may immediately find your share of the profits. Your brother gets what is left, and if you wish to know it, you then may use equation 1, and subtract your profit from the total of \$1500 profit.

23. The T.S. Tire Shop bought 120 of the manufacturer's special wide tires, but they were selling slowly and a new type was being introduced soon. So they advertised a weekend sale for the remaining inventory, reducing the regular price of \$20 each to just \$10, on Saturday only. The tires were sold and the manager found that the total sales receipts for all 120 tires were \$1700. How many tires were sold on Saturday?

24. Let's write the relationships. We know that Saturday's tire sales, say S, plus previous sales, P, total 120 tires. What else do we know about S and P? ($S=\frac{120P}{1700}$) ($\frac{S}{10}=\frac{P}{20}$) (10S+20P=1700)

25. Right. Now for the solution of these equations. How about dividing everything in this second equation by 10? This gives us S+2P=170. What would you suggest doing now? (substitute 120-P for S in equation 2) (multiply both equations by 2, then add 170)

26. Yes. That's one way to solve it. This gives us (120-P)+2P=170. Dropping the parentheses, which we can do because there's no multiplication or division indicated, we do the indicated operations first getting 120+P=170; then what should we do? (subtract 120 from both sides; P=50) (divide both sides by 120; $P=\frac{170}{120}$)

27. Yes, and if P, the number of tires sold previously at \$20, equals 50 tires, then S, the number of tires sold at \$10, is what? ($S=\frac{1700}{50}-120$; S=64) (S=120-50; S=70)

28. Right. One way to measure wind velocity is to measure the speed of sound along a line between two microphones with a source of sound between them. Dr. Goldstein put two small radio transmitters with microphones 100 meters apart in line with the wind direction, and connected one earphone to each of the receivers.

29. Then he walked between the microphones with a clapstick until he heard the sound coming from both receivers at the same instant. If the speed of sound was 300 meters per second, and Dr. Goldstein was 48 meters from the nearest microphone, what was the wind velocity?

30. Let's see, net sound velocity is the distance divided by time in meters per second; and it is 300±W, which stands for wind. The time T is the same for both distances, 48 and 52 meters, but the net velocity of the sound for the 48-meter distance was different. What was it? (W) (300+W) (300-W)

31. Right. It was slower, because it travelled a shorter distance. We could say then, that this velocity

equaled 48 divided by the time required, T, whatever it was. So $300\text{-}W=\frac{48}{T}$. What else do we know? [$T=300+W(48)$] [nothing] [$300+W=\frac{52}{T}$]

32. Right again. Now here are our two equations. $300\text{-}W=\frac{48}{T}$ and $300+W=\frac{52}{T}$. If we were to add these two equations, would we eliminate the unknown, "W"? (Yes) (No)

33. Yes. We get $600=\frac{48}{T}+\frac{52}{T}$. What would you suggest now? (multiply both sides by T) (subtract 600 from both sides) (take a coffee break)

34. Yes. And we'd get $600T=48+52=100$, and $T=\frac{1}{6}$ second. How do we find W now? (substitute $\frac{1}{6}$ for T in $300+W=\frac{52}{T}$) (multiply T times 300+W, add 52)

35. Yes. This would be $300+W=\frac{52}{1/6}=6$ times $52=312$. What is the wind velocity? (48 meters per second) (52 meters per second) (12 meters per second; 27 mph)

36. Right, but there probably is a better way to measure simultaneous sounds than Dr. Goldstein's ears. Right now, let's learn two new symbols of algebra, and we'll work with them later. First, what does this symbol mean? [=] (is less than) (is equal to) (is greater than)

37. Yes, of course. If you write this, you would read 1 "is less than" 2. What does this symbol mean? [<] (is less than) (is equal to) (is greater than)

38. What would you think that this means? [>] (is less than) (is equal to) (is greater than)

39. Which symbol means "is less than"? (<) (=) (>)

40. Which symbol means "is greater than"? (<) (=) (>)

BASIC ALGEBRA Reference Folder Ma 13

Exponents

1. There are many occasions when quantities are multiplied by themselves. When a number has been multiplied once by itself, instead of writing it as an operation to be performed, we usually write its "square" value, if it is a number; or if it is a quantity represented by a letter, we show the small exponent 2 at its upper right. Instead of X times X times X, we show X cubed, with a small 3 as its exponent. The 3 means that there were three identical quantities to be multiplied, but there were only how many multiplications to be performed? (2) (1)

2. If a quantity is an unknown represented by a letter with a number as a coefficient in the term to be multiplied by itself, you will multiply coefficients as well as letters. If we had a square 3X inches on each side, its area would be $9X^2$. What's 4Y times 4Y? ($16Y^2$) ($4Y^2$)

3. When we have a cube whose area on one face is S^2, its volume can be obtained by multiplying this area once more by the side length S. What is S^2 times S? (S^3) ($2S^2$) (S^4)

4. We know that X times X is X^2, and X times X times X times X is X^4. What is X^2 times X^2? ($2X^2$) (X^4)

5. Right. But not because we multiplied the exponents, but because we added them. You multiply coefficients, but you add the exponents to obtain the product of a quantity times itself. You must remember, of course, that there is an understood 1 for the coefficient and an understood 1 for the exponent of a quantity such as X in the binomial (X^2+3Y^2). Do not confuse 1 and 0 as exponents, for X to the first power=X, whereas X to the zero power is just one.

6. What is the product of Z^2 times Z^3? (Z^5) (Z^6)

7. What is the quantity ($2XY^3$) times itself, that is, squared? ($4X^2Y^5$) ($4X^2Y^6$)

8. Right. Another way of writing this is $(2XY^3)^2$. In this case, you don't add the exponents 3 and 2, because this is a shorthand way of saying that $2XY^3$ is multiplied by $2XY^3$. So you are really adding the two Y exponents, or adding 3 to 3, to get Y to the sixth power.

9. In this case, you might say that the exponent outside the parentheses, applying to the entire term inside the parentheses, is multiplied by each inside exponent. What would be a simplified form of the term "three A squared, B to the fourth power, closed in parentheses and the entire term raised to the third power, or cubed?" ($9A^5B^7$) ($27A^6B^{12}$)

10. Right. What is the product of $2X^2Y^3$ times $3XY^2$? ($6X^3Y^5$) ($5X^2Y^6$) ($6X^2Y^6$)

11. Yes. What is the term (P^2Q^3) raised to the fourth power? (P^6Q^7) (P^8Q^{12})

12. Right. Don't hesitate to repeat this part of the program if you need to review the rules. You will

remember from your arithmetic studies, or from the program Mg6, what the radical sign means, and what a square root or cube root is. What do you think a radical means to a mathematician? (a wild-eyed long-hair) (distance from center to outside of circle) (the inverse of an exponent)

13. Right. Actually, the term radical means root-like, or basic, and in common language a person who wishes to make radical reforms would like to make basic or thorough changes.

14. To the mathematician, a quantity, called a root, multiplied by itself the indicated number of times, gives as a product the number under the radical sign. The radical sign, of course, merely signifies the inverse operation of finding that root.

15. For example, a radical with a small 2 above it means the quantity under the radical sign is the product of some number multiplied once times itself. With no number at all,it is naturally assumed to be a 2. If there is a small three called the "index" above the radical, it is a cube root, and the quantity under the radical is the product of a number multiplied by itself twice. 2X times 2X times 2X is 8X cubed. The cube root of 8X cubed is what? (2X) (8X) ($2X^2$)

16. Yes. The cube of -2X, that is, -2X "raised to the third power," would be $-8X^3$, and the cube root of $-8X^3$ is -2X, of course. This is because -2X times -2X is $+4X^2$, and $+4X^2$ times -2X again is $-8X^3$. But what is the square root of $4X^2$? It could be either +2X or -2X! We'll just say it's + or -2X, we don't know which, and use the ± sign.

17. To find, obtain, or extract the root of a single term, you first find the root of the numerical coefficient. This may be done by using a calculator or from memory. The exponent of every quantity represented by a letter is then divided by the index number on the radical. Thus $\sqrt[3]{8X^6}$ is $2X^2$ and $\sqrt[3]{27X^{15}}$ is $3X^5$. What is the square root of $25X^6$? ($\pm 5X^3$) ($5X^4$) ($-5X^3$)

18. Yes. What is the square root of $40X^4$-$4X^4$? ($6X^2$) ($36X^2$) ($\pm 6X^2$)

19. Right. By the way, here's a quick way to get the square root of an arithmetic number using a small electronic calculator which doesn't have a square root key. First enter the number, then guess at the square root and divide by your guess. Look at this quotient, mentally pick a number between your original guess and this quotient, and divide the number again, this time by your estimated average. To get closer, then, mentally or with the calculator, average your second guess with the second quotient. This should be pretty close, but you can repeat this step again if you want to carry it out to several decimal places. What's the square root of 256? (25.4) (26) (16)

20. Right. You should remember the way binomials are multiplied together. What is $(X+3)^2$? (X^2+3) (X^2-3) (X^2+6X+9)

21. Right. The square of any binomial is a trinomial, because the cross-products of the two terms in the binomial are the same, and they never cancel out. This means that only a trinomial can have a regular square root, and it's a binomial.

22. The trinomials which are perfect squares always take the form of $A^2+2AB+B^2$. You could easily take the square root of, say, $C^2X^2+2CDXY+D^2Y^2$, because it's just CX+DY. What's the positive square root of $9X^2+12XY+4Y^2$? (3X+2Y) (3X-2Y) (X+6Y)

23. Yes. We omitted the negative root. You may be confused by this use of the word "root," as our previous use of the word was the solution of an equation. In a sense, they both mean the basic result or the underlying or supporting quantity.

24. Here is a part of a table of Powers and Roots. It gives the square, cube, square root and cube root of numbers from one to 100. What is the cube root of 8? (1.913) (2.000) (2.080)

25. Yes. In many instances you don't need to show all the decimal places. However,it's sometimes helpful if you have a calculator to carry an extra digit for accuracy in rounding off. But in the final answer, excessive places and digits may imply an accuracy in the original information which is probably not true, and so would be misleading. You may need to round off in order to avoid misleading answers.

26. In program Mp6,you learned how to "round off" numbers or eliminate meaningless or misleading places and digits. In general, most quantities except money are only meaningful to one part in 100, or perhaps one part per thousand; that is,two to four digits, since most everyday accuracy doesn't exceed this. For precision machinery, surveys, or astronomical observations, of course, we need more digits.

27. What is 123.456 to the nearest tenth? (123) (123.4) (123.5)

28. What is 123.456 to the nearest hundredth? (123.5) (123.46) (123.56)

29. You will recall from program Mg9 the Pythagorean theorem about right triangles. It states that the square of the hypotenuse is equal to the sum of the squares of the other two sides. This is rather easy to do on your calculator, although on the smallest ones you will need a scratch pad. What's the square root of the quantity "3 squared plus 4 squared"? (5.00) (6.00) (7.00)

30. Yes. If you know that the area of a right triangle is 5 square feet, and that one of the legs, a side next to the 90° angle, is 3 feet longer than the other, how long is it? Let's see, if the legs are X and Y, X=Y+3. What else do we know? ($\frac{1}{2}(XY)=5$) ($XY=5$) ($X^2+Y^2=5$)

31. Yes. The area of a right triangle is just half of the area of a rectangle with its sides equal to the triangle's legs, as you can see. Now we have two equations,X=Y+3 and ½XY=5. This second equation is not a simple linear equation. If we were to graph it, we'd get a curved line. But we can solve these two equations by simple substitution. What would we get by substituting Y+3 for X in the second equation? ($\frac{1}{2}(Y+3)(Y)=5$) ($\frac{1}{2}(X^2)(Y^2)=5$)

32. Yes. Now we perform the two indicated multiplication operations for the left side. What do we get? ($\frac{3}{3}Y^3=5$) ($\frac{1}{2}Y^2+\frac{3}{2}Y=5$)

33. Right. It might have been easier to multiply the binomial by Y, then multiply both sides of the equation by two to get rid of the fraction. Let's do it now. What do we get? ($Y^2+3Y=10$) ($2Y^2+3Y=10$)

34. Yes. Now, can we just add Y^2 and 3Y to get $4Y^2$? (Yes) (No)

35. No, we will just have to work with this equation as it is. The standard form would be $Y^2+3Y-10=0$. Let's see. Can the trinomial expression on the left side be factored? (Yes) (No)

36. Yes, it can. What do you think its factors are? [(X+3), (X-10)] [(X+5), (X+2)] [(X+5), (X-2)]

37. Right. And at this point, you can tell that the triangle has legs that are two feet and five feet long.

38. In an earlier program, irrational and imaginary numbers were mentioned. Now we will use them in dealing with roots. Notice that the word "irrational" has buried in the middle of it the word "ratio." An irrational number is not the ratio of any ordinary number to another number, meaning it is not divided by the number. The square root of any number which is not a perfect square, (and most numbers are not perfect squares), is an irrational number. Is the square root of 6 an irrational number? (Yes) (No)

39. Yes. Some numbers besides square roots and other roots are not ratios and, therefore, are irrational. Which of these do you think is probably irrational? ($\sqrt{4}$) ($\sqrt[3]{8}$) (π)

40. Right. An imaginary number is the square root, or other even numbered root, of a negative number. The square root of minus 4 is not 2, or even -2. Which of these would you guess is the square root of -4? [$-2(\sqrt{-1})$] [$4(\sqrt{2})$]

41. Yes, and sometimes mathematicians can work with this square-root-of-minus-one term. They use a lower case i to represent it. What do you think this i represents? (imaginary) (irrational) (irritation)

42. Yes. Don't forget, you should repeat this lesson several times to be sure you understand and remember all of the rules of algebra you have learned.

1. In the previous program you learned about numbers multiplied times themselves, or "raised to second, (or third) powers," and so on. Numbers which are squared, cubed, or raised to the fourth or fifth power, for example, are used to solve real problems today, and they occur in algebraic relationships. What do we call an algebraic statement of relationships between quantities? (an equivocation) (an equation) (an equestrian)

2. Good. An equation which contains terms up to the second degree, that is, "squared" terms, is called a "quadratic" equation. Now you probably know that the Latin word "quadrant" means "fourth" and "quads" generally refer to things with four elements. The reason mathematicians say "quadratic" for only second degree equations is based on area matters, usually involving "quadrangles" or rectangles. Area, of course,is a two-dimensional concept which requires squared or second-order terms.

3. "Quad" means "four;" "quadrant" means fourth; but the highest terms in a quadratic equation are what? (first degree) (second degree) (fourth degree)

4. Correct. We have actually solved some second-degree or quadratic equations in earlier programs by inspection, or by factoring. We will get some more factoring practice in this program. First, however, we'll review the standard form of a quadratic equation.

5. This standard form is arranged with the terms in descending order on the left side of the equation, and with zero on the right side. Thus,the first term is a second order term, say, $3X^2$; the next is the first order term, say, $7X$; and the third is a constant, say, 2. This trinomial is set equal to zero. Which is the standard form of $5X=12-3X^2$? ($3X^2+5X-12=0$) ($3X^2=12-5X$)

6. What is the standard form of $X^2=9$? ($X^2+9X+9=0$) ($X^2-9=0$)

7. Yes. Sometimes a quadratic equation doesn't have a first order term in the unknown at all. You might say it is there in spirit, but its coefficient is zero, so we don't see it. Also,we may have no constant term, or it may be considered zero, so we don't write it. What's the standard form of $X^2=3X$? ($X^2-3X=0$) ($X^2-X+3=0$)

8. Yes, but in this case the X could be factored out, and one root would be $X=0$. The remaining equation root, for practical purposes, might be considered linear, or first-order equation.

9. Let's call the general expression of the standard form of a quadratic equation $AX^2+BX+C=0$. A, B, and C are any given numbers,such as $2X^2+3X+4=0$; therefore, A, B, and C are 2, 3, and 4 respectively. If you had an equation $5+7X=-X^2$, which coefficient number is the "B" in the standard form? (5) (7) (1)

10. Yes. As you know, when the "B" coefficient of the first-order term is 0, this term is missing, such as in $X^2-9=0$. This makes the equation easy to solve, since in the form $AX^2+C=0$, $X^2=\frac{-C}{A}$ after subtracting

from both sides C, and dividing both sides by A. Then what does X equal? $(X=\sqrt{\frac{C}{A}})$ $(X=\pm\sqrt{\frac{-C}{A}})$

11. Right. Of course, if either C or A is a negative number, you will get an imaginary root. In all of our problems we will deal with real numbers.

12. In X^2-9=0, the left side can be factored into (X+3) times (X-3), so we know that X is equal to both -3 and +3. We could also solve for X by adding 9 to both sides to get X^2=9; then we see that X equals the square root of 9. What's $\sqrt{9}$? (3) (4½) (± 3)

13. Right. And if we have the equation $4X^2$-9=0, we'd solve for X^2 getting $X^2=\frac{9}{4}$. Then take the square root of both sides. What do we find? $(\frac{9}{4})$ $(4\frac{1}{2})$ $(\pm\frac{3}{2})$

14. Right. Most of the time, however, we'll have all three kinds of terms in our quadratic equation—the second order, the first order, and the constant. And in this general case, we should look quickly at the coefficients to see if they will probably permit simple factoring. For example, with two binomials AX+M and DX+N multiplied together, we will get ADX^2+(AN+DM)X+MN. Look at this carefully. Do you think you could factor $6X^2$+22X+20? (looks likely) (seems impossible)

15. Yes. A and D could be 2 and 3; and M and N could be 4 and 5. In fact, $6X^2$+22X+20 can be factored into (2X+4) and (3X+5). The sum of the cross products of 2X times 5, plus 3X times 4, does give 22X for the middle term. We could go on with tricks in factoring, but there is a formula which you can memorize easily which solves second-order equations routinely. What would you guess this formula is called? (binomial formula) (triatic formula) (quadratic formula)

16. Right. But first, a few frames of factoring practice. What are the roots of X^2+5X+6=0? (X=-5; X=-6) (X=-2; X=-3) (X=2; X=3)

17. Yes. The binomial factors are X+2 and X+3. This product is equal to zero, so either X+2 is 0, or X+3 is 0. If X+2 equals 0, we subtract 2 from both sides, and X equals -2. When we solve or clear up the equation for the unknown letter-quantity by subtracting the constant from both sides, we find that constant on the other side with its sign changed.

18. What are the roots of X^2-9X+20=0? (X=4; X=5) (X=3; X=10) (X=-4; X=-5)

19. Yes. The trinomial expression factors into (X-4) times (X-5), and it equals 0, so your answer-giving roots of 4 and 5 are correct. Now let's learn the quadratic formula which can be used to solve any quadratic equation.

20. The standard form of a quadratic equation is AX^2+BX+C=0. To get the roots of X,you divide the quantity -B ± the square root of B^2-4A times C, allover 2A. Perhaps you can set it to some jingle, as a mnemonic trick, like minus B plusorminus the square root of B^2-4AC, allover 2A. Minusbee plusorminus square root of B^2-4AC allover 2A. You may need to repeat this several times before you go on.

21. How about a little memory drill? The quadratic formula tells us that the roots of X in the equation AX^2+BX+C=0 are. . . minusbee-plusorminus-squarerootof-B-squared-minus-4AC-allover. . what? (AC) (2A) (2B)

22. Yes. Now let's try out the formula on X^2-5X+6. A is 1, B is -5, and C is 6, so X equals $-(-5)\pm\sqrt{25-4(1)(6)}$ all over 2. When we do the indicated multiplications and subtractions, we get . . . what? $(X=\frac{+5\pm\sqrt{1}}{2})$ $(X=\frac{-5\pm\sqrt{4}}{2})$

23. Yes. Since the square root of one is ± 1, we have both $\frac{5+1}{2}$ and $\frac{5-1}{2}$ as roots. What are these two quantities? (X=3; X=2) (X=4; X=5)

24. Yes. Now try $2X^2-X=1$. This is not in standard form; it should be $2X^2-X-1=0$. A is 2, B is -1, and C is -1. Then what do we have for a substitution in the formula? $(X=\frac{1\pm\sqrt{1-4(2)(-1)}}{2(2)})$ $(X=\frac{-1\pm\sqrt{1-4(2)(1)}}{2(2)})$

25. Yes, and evaluating $\frac{1\pm\sqrt{1-4(2)(-1)}}{2(2)}$, we get $\frac{1\pm\sqrt{1+8}}{4}$ or $\frac{1\pm\sqrt{9}}{4}$ which is $\frac{1\pm3}{4}$. What are the roots? (X=-½; X=2) (X=-1; X=-2)

26. Quite right. As you can see, all you need to do is to write the equation in standard form, substitute A, B, and C in the quadratic formula, and do the indicated operations. In general, if the quantity under the radical symbol, (called the discriminant), is a perfect square, like 9, 16, or 25, you should go ahead and do all the arithmetic operations. If it's some irrational number, you may usually show your solution by just leaving the radical in the answer. You won't be given a problem with an imaginary answer, that is, with a negative quantity under the radical.

27. Since you know the multiplication table, you already know that 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100 are perfect squares. It may help to memorize that 121, 144, 169, 196, 225, and 256 are perfect squares for 11 through 16, and 625 is the square of 25. Naturally, you recognize that 100, 400, 900, 1600, 2500 and so on are the squares of 10, 20, 30, 40, and 50. After you have studied these numbers, guess what the square root of 484 is. (18) (22) (28)

28. Yes. You may have access to a small electronic calculator, so you'd think that it would hardly be necessary to do all this drill and memory work. There are several reasons: some test examiners may not permit calculators and prefer that you use tables, mental work can be more satisfying, and if you make a mistake in operating your calculator, a rough mental check may reveal it.

29. If we had a quadratic equation given as $10X-X^2=25$, how would we use the formula? First, the standard form: arranging terms in proper order, we get $-X^2+10X-25=0$, but for convenience let's multiply through by -1 and get $X^2-10X+25=0$. As a good student, you may have already guessed at the roots, but let's go through the formula.

30. The values of A, B, and C are 1, -10, and 25. The formula gives us $X=\frac{10\pm\sqrt{100-4(1)(25)}}{2(1)}$ and we see immediately that the discriminant or the number under the radical is 0. Therefore, what are the roots? (X=5; X=5) (X=5; X=10) (X=10; X=10)

31. Yes. You probably knew that from your inspection of the standard form, your practice with factoring, and your memory of perfect squares. Here's a quadratic problem: Bob and Bill each agreed to mow half of the lawn, which is 90 by 120 feet. Bob mowed around the outside of the lawn, mowing a wider and wider strip. How wide was it when he stopped to let Bill mow the rest?

32. Let's see, the inside rectangle is half as large in area as the entire rectangle, which is 90 by 120 feet. Now we can call the mown strip X feet wide. Then the area of the inside rectangle is the product of its remaining length and width, or (90-2X) times (120-2X). This remaining area is also half of 90 times 120, or $\frac{90 \times 120}{2}$. Now we have an equation. What is it? [$\frac{(90)(120)-4X^2}{2}=90+120$] [$(90-2X)(120-2X)=\frac{(90)(120)}{2}$]

33. Right. Now let's perform the indicated operations; multiplying the binomials gives us $(90)(120)-(90)(2X)-(120)(2X)+4X^2=\frac{(90)(120)}{2}$. Collecting terms and putting them into order, we get $4X^2-420X+10{,}800=5400$. That's still not standard form; we need a zero on the right side, so we subtract 5400 from both sides and get what? ($4X^2-420X+5{,}400=0$) ($4X^2+120X-5{,}400=0$)

34. Yes. And just for further simplification, let's divide everything by 4, and get $X^2-105X+1{,}350=0$. Which is the quadratic formula applied to this equation? ($X=\frac{105\pm\sqrt{105^2-4(1{,}350)}}{2}$) ($X=\frac{1{,}350\pm\sqrt{1{,}350^2-4(1)(105)}}{2(105)}$)

35. Correct, and this is $X=\frac{105\pm\sqrt{11{,}025-5{,}400}}{2}$. After subtracting, we find the discriminant is $\sqrt{5{,}625}$. What do you think this equals? (45) (55) (75)

36. Right. 105 ± 75 divided by 2 is either 90 or 15. Obviously, Bob isn't going to mow a strip 90 feet wide around a yard that is 90 by 120 feet, so he stops when he makes enough circuits to mow a 15-foot belt around the yard.

37. This would leave a 60 by 90 foot remainder of uncut grass in the center, which is half of the 90 by 120 foot yard, of course. Who mows the remainder? (Bob) (Bill) (Nobody)

38. A bomber airplane flew at a constant airspeed to a target 1800 miles away against a jet stream headwind of 100 miles per hour, but flew back with it as a tailwind. The round trip took 6 hours. What was its airspeed? It's not exactly 3600 miles divided by 6 hours, or 600 miles per hour, but it's somewhat over this speed, because the headwind trip took longer than the tailwind return. The wind caused a slight net loss of time. Let's do the algebra. If the airspeed is X, the outbound trip took $\frac{1800}{X-100}$ hours and return trip $\frac{1800}{X+100}$ hours, both trips totaling 6 hours. The equation then, should be. . . what? ($\frac{1800}{X-100}+\frac{1800}{X+100}=6$ hours) ($\frac{1800+1800}{X+100-100}=6$ hours)

39. Yes, but this is hardly in standard form. Let's see what we can do. Multiplying both sides by (X-100)(X+100) we would get $1800(X+100)+1800(X-100)=6(X+100)(X-100)$, then we need to perform indicated multiplications and additions. We then can transfer by subtraction to get the standard form. Would you say that $1800X+1800X=6(X^2-10{,}000)$ is a correct intermediate step? (Yes) (No)

40. Yes. Now to simplify a little, let's divide everything by 6, add the X terms, and get $600X=X^2-10{,}000$. With a little subtraction, and multiplying by -1, we get $X^2-600X-10{,}000=0$ in the standard form. Applying the quadratic formula, $\frac{+600\pm\sqrt{-600^2+4(10{,}000)}}{2}$ equal to $\frac{600\pm\sqrt{360{,}000+40{,}000}}{2}$. You can see this will be over 616 miles per hour. Thus, the bomber will require a little more jet fuel than in quiet air. We'll hope they took some to spare. In the next program we'll look at how quadratic equations can be used to graph curves.

41. Don't forget, you should repeat this lesson several times to be sure you understand and remember all of the rules of algebra you have learned.

Dorsett Educational Systems, Inc. Box 1226 Norman, Oklahoma 73070 (405) 288-2300

BASIC ALGEBRA Reference Folder Ma 15

More Quadratics

1. You know that second-order, or quadratic, equations have a term in which the unknown quantity appears at the second order (and no higher); that is, "squared" as in $3X^2$ or $4Y^2$, as denoted by the small exponent 2. This means the unknown has been multiplied by itself once.

2. A quadratic equation must have at least one more term besides the second-order term, of course, or it's not an equation. If it only has one more term, it may be the number-only term, in which case the unknown is easy to find; it is just the square root of the number term after being divided by the second-order term's coefficient. What does X equal in this equation? $2X^2=18$ ($X=\pm3$) ($X=36$) ($X=9$)

3. Yes. If there are only two non-zero terms in a quadratic equation, and one is a linear, or first-order term of the unknown, then one root is, of course, just zero. For example, if $2X^2=10X$, you could satisfy the equation once with X being zero. In the standard form $AX^2+BX+C=0$, you might say that $2X^2-10X+0=0$ and the C or constant number is zero. So when there is no constant term in a quadratic equation, what is true? (The variable is unknown.) (The constant is variable.) (One root is zero.)

4. Right. You may then divide both terms by the unknown and have a linear equation, but in doing this, don't forget the zero root. What are the two roots of $2X^2=10X$? ($X=0$; $X=5$) ($X=2$; $X=10$) ($X=2$; $X=5$)

5. Yes. We could say that a quadratic equation is in its complete form if it has three terms when combined, if necessary: the second-order term, the linear or first-order term, and a non-zero constant term. Otherwise, it's in an incomplete form.

6. A hectare is an area of about 2½ acres, which equals a square 100 meters on each side. If you bought some rectangular-shaped property in Mexico totalling 6.4 hectares, and it was only one-fourth as wide on the front side facing the highway as its distance to the rear boundary, what size was it? First, how many square meters are in it? (640) (2500) (64,000)

7. Right. And we can say that X times 4X is 64,000 square meters; then $4X^2=64{,}000$ is our equation. What kind of quadratic equation did we call this? (complete) (linear) (incomplete)

8. Right. A quadratic equation in complete form has three terms, and in standard form, you recall, it is $AX^2+BX+C=0$. And in the previous program, you remember that complete quadratic equations can be solved in two ways. What are they? (transposition, radical change) (factoring, quadratic formula) (substitution, elimination)

9. Yes. For example, which way would you be likely to solve this equation? $X^2-6X+9=0$ (factoring) (quadratic formula)

10. Yes, the two roots would both be +3, since $X^2-6X+9=0$ factors to $(X-3)(X-3)=0$, and the equation is

satisfied by X=3 and X=3.

11. But you might be tempted to try the quadratic formula on, say, $10X^2+13X-9=0$, unless you were pretty skillful at factoring. Let's review the quadratic formula: "minusbee plusorminus squarerootof beesquared minus-fourAC over 2A." Of course, you've drilled on this a lot. How will this equation look when plugged into the formula? $(X=\frac{-10\pm\sqrt{10-10(9)}}{2(9)})$ $(X=\frac{-13\pm\sqrt{169+4(10)(9)}}{20})$

12. Right, and this reduces to $\frac{-13\pm\sqrt{169+360}}{20}$, or $\frac{-13\pm\sqrt{529}}{20}$, or $\frac{-13\pm 23}{20}$. What is one of the roots? $(X=\frac{1}{2})$ $(X=\frac{13}{20})$ $(X=1)$

13. Correct. Now look again. What's the other root if $X=\frac{-13\pm 23}{20}$? $(X=\frac{9}{5})$ $(X=\frac{23}{10})$ $(X=-\frac{9}{5})$

14. Right. There's really nothing to it, is there? Just get the equation into standard form, remember the formula, and plug A, B, and C into the right places. You do have to be rather careful with your arithmetic, but fortunately most of the problems you must do for tests you take will be simple numbers, and most of the algebra with odd numbers that you perform on the job will be done by computer or calculator.

15. Let's try another easy one. How about $X=12-6X^2$. What do we do first? (divide both sides by 6) (subtract 12X from left side) (put equation in standard form)

16. Right. And this actually means getting zero on the right side and all the terms, in their proper order, on the left side. What do we get? $(6X^2+X-12=0)$ $(-6X^2+12X-1=0)$

17. Yes. What, respectively, are A, B, and C? (6, 1, and 12) (6, X, and 12) (6, 1, and -12)

18. Right. And with A, B, and C being 6, 1, and -12, what does the formula look like? $(\frac{-1\pm\sqrt{1-4(6)(-12)}}{2(6)})$ $(\frac{-1\pm\sqrt{36-4(1)(-12)}}{2(12)})$

19. Yes. Now let's do the arithmetic quickly. Pick the correct simplification. $(\frac{-1\pm\sqrt{1+288}}{12})$ $(\frac{-1\pm\sqrt{1-288}}{12})$

20. Yes. Now the next step is $\frac{-1\pm\sqrt{289}}{12}$. What's the square root of 289? (27) (14.7) (17)

21. Right. And now we get the two roots from $\frac{-18}{12}$ and $\frac{16}{12}$. What are they? $(X=\frac{-3}{2}, X=\frac{4}{3})$ $(X=\frac{3}{2}, X=\frac{4}{3})$ $(X=\frac{3}{4}, X=\frac{2}{3})$

22. Yes. Of course, you could have gotten these roots from factoring $6X^2+X-12$ by a little strategy like this: what two factors of 6, and what two factors of -12, when cross multiplied, will algebraically total 1, which, of course, is the linear coefficient B. Try 3 and 2 as the factors of 6, and minus 4 and 3 as the factors of minus 12; cross multiply to get minus 8 and 9 respectively, and their difference is one—the constant C. Tricky, isn't it? (Yes) (aw, not very) (No)

23. Yes, and having factored the equation into 2X+3 and 3X-4, you could see pretty quickly that X is $\frac{4}{3}$ and $\frac{-3}{2}$.

24. You've already learned that a first-order, or linear, equation will graph as a straight line. Equations, which have second-order, (squared), terms or higher order, will graph as what kind of lines? (angular) (square) (curved)

25. Right. Here's a graph of the equation $Y=6X^2+X-12$. It's the same as the one we just solved except in this case the quadratic expression, made up of constant and X terms, is not set equal to zero, but to a variable Y. We say that Y is a function of X, and you can see that Y, the vertically plotted value, is a fairly complex, curved function.

26. The curve you get from any standard quadratic equation like $AX^2+BX+C=Y$ is called a parabola, which is an extremely common curve occurring in flashlight reflectors, in automobile headlights, and bullets. Of course, depending on the values of A, B, and C, it may be fatter or thinner or just include a portion of the end of the nose.

27. Here's a graph of $Y=X^2-4$. You can see by looking at the equation that when X is 0, Y is -4, and that when X is either +2 or -2, Y is 0. Notice these three points on the parabola which is graphed. What do you think the value of X would be when Y is -5? (X=2½) (X=5) (never happens)

28. Right. Y is a function of X. We can assign any plus or minus value to X that we choose and get a value for the Y, which is a real function of X. But none of these values of Y, the function of X, is less than -4 in $Y=X^2-4$. Why is this? (All values of X^2 are positive (or zero).) (Y is not functioning.) (4 is the square of 2.)

29. Yes. Since X^2 is always a positive number, no matter what value X assumes, there may be values of Y, a quadratic function of X, which never occur.

30. If you wanted to make a hardware bin from a 9 x 12 inch piece of sheet metal, and you needed a base area of 60 square inches, how high would the folded up sides be if you make them all equal? In other words, what size would the square corner notches be?

31. Before we start, let's try a quick mental trick to guess at the approximate size of the completed bin or tray. If we needed 60 square inches, we could make the base 6 inches wide and 10 inches long. But this would make the folded up sides one inch high on the ends from the 12-inch piece and 10-inch base, and one and one-half inches high from the original 9-inch width and finished base of the bin 6 inches wide. Can you guess about how high the sides will be on the bin we actually make? (1 inch) (about 1¼ inches) (1½ inches)

32. Yes, you've guessed pretty close to the answer already. But now let's do the algebra, which involved a quadratic equation, of course. Let X be the height of the sides. How would you express the 60-square-inch base area as a function of X? [$X^2=60+3^2$] [$(9-2X)(12-2X)=60$] [$60=X^2+9+12$]

33. Yes, the base area is the product of the base width and length, which is the product of 9 and 12 after subtracting the folded sides. Next we perform the indicated operations and put the equation into standard quadratic form. What do we get by multiplying the binomials (9-2X) and (12-2X)? ($108-42X+4X^2=60$) ($108+24X+4X^2=60$)

34. Yes. Is this in standard form? (Yes) (No)

35. Right. It wasn't. How about this? $4X^2$-42X+48=0 (standard form) (funky form)

36. Yes. Now for a memory drill. Which of these is in the quadratic formula? ($X=\frac{B\pm\sqrt{B^2-4AC}}{A}$) ($\frac{B\pm\sqrt{B^2-4AC}}{2A}$) ($\frac{-B\pm\sqrt{B^2-4AC}}{2A}$)

37. Right. Here's our equation. Which of the choices below shows the correct use of the quadratic formula? ($\frac{42\pm\sqrt{(42)^2-4(4)(48)}}{2(4)}$) ($\frac{-42\pm\sqrt{(42)^2+4(4)(48)}}{2(4)}$)

38. Yes. Now to punch out a few buttons on our hand calculator, we get 42 squared to be 1764, and 4 times 4 times 48 to be 768, so we have 996 under the radical. What would you guess is the square root of 996? (30) (31.5) (33)

39. Close enough. This gives us either 73½ over 8, or 10½ over 8. We know that the X we want is about 1 inch, so we'll settle for what? ($1\frac{1}{8}$) ($1\frac{1}{4}$) ($1\frac{5}{16}$)

40. Right. The other root seems to be a rather odd way of folding up the bin, so we'll ignore it. You can see that in computing marginal areas and sizes, algebra is quite useful. What do you suppose Hirozo Tanaka, the engineer at Sony, used in calculating the changes in color TV screen area? (bamboo shoots) (old Japanese proverb) (quadratic equations)

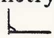
BASIC ALGEBRA

Reference Folder Ma 16

Equations in Trigonometry

1. You've heard of the mathematical term trigonometry, and from its name, you can guess that it's related to geometry and deals with angles and triangles. A triangle, you know, is an enclosed geometric figure with three straight sides. It has three other features, too. What are they? (three angles) (three diagonals) (three bases)

2. Yes, of course. And these angles may be of several kinds, like acute angles, right angles and obtuse angles. Which of these is an acute angle? () () ()

3. Yes. For more review on geometry, study the Mg series of programs. Do you remember which of these is a right angle? () () ()

4. Yes. A right angle is formed by the intersection of two perpendicular lines which form four equal angles. Mathematicians have chosen to say that a right angle contains 90 degrees, so an acute angle has less than 90 degrees and an obtuse angle has more than 90 degrees. The four right angles, formed when two perpendicular lines cross, add up to how many degrees? (300°) (360°) (400°)

5. Yes. You probably recall several things about triangles. The point at the angle where the lines meet is the vertex. A triangle has three vertexes, or vertices. The total of the angles in a triangle is 180 degrees, but in a rectangle it's 360 degrees. In a right triangle, (a triangle where one of the angles is 90°), the total of the other two angles, naturally, is 90°. Can there be two right angles in a triangle? (Yes) (No)

6. No, not quite. Perhaps you remember the name of the longest side, the diagonal side opposite the 90° angle in a right triangle. It's called the hypotenuse. What do you think the two sides next to the right angle are called? (legs) (arms) (tails)

7. Yes, the legs of the right triangle. Usually we designate the sides of a right triangle with lower-case letters corresponding with upper-case letters indicating the angle opposite that side. This means that if capital A and capital B are the two acute angles of a right triangle, small a and b are the two legs, or shorter sides, and then capital C would be the right angle. What would lower-case c designate? (the right angle) (a leg) (the hypotenuse)

8. Right. If you have studied program Mg7 recently, you recall the Pythagorean theorem, which states that the length of the hypotenuse is the square root of "a" squared plus "b" squared. Another way, is to say that "the square of the hypotenuse equals the sum of the squares of the legs."

9. For the purpose of many statements and examples in trigonometry, we'll consider a standard right triangle with angles at A, B, and C. The right side is vertical and the base is horizontal, so C is a right angle. What do we know about angle A? (it's always 45°) (it's always less than B) (it's always less than 90°)

10. Right. If we draw another vertical line, say D to E, we'd make another included triangle. We know from Ma 10, that there is a proportion, or equality of ratios, between the ratio of the line length AC divided by AB, and the ratio AE over AD. This proportion always occurs in similar triangles. What are the triangles that have the same angle A, and each have a 90° angle? (similar) (different)

11. Of course. This constant ratio of the base, or leg adjacent to angle A, divided by the hypotenuse, is called the "cosine" of the angle. Sometimes the word "cosine" is abbreviated as "cos" in lower case. How would you describe the cosine of angle A? (ratio of length of adjacent leg to the hypotenuse) (ratio of the angular size of A and C)

12. Yes. Another trigonometric ratio is the "sine," which is the ratio of the length of the leg in the reference triangle opposite to the reference angle A, to the length of the hypotenuse. Notice that this word "sine" is spelled differently from the ordinary word "sign." Sometimes it is abbreviated "s-i-n", but there's nothing sinful about it. It's just a ratio.

13. Which of these is the ratio of the opposite leg over the hypotenuse? (sign) (sine) (cosine)

14. A third important ratio is the "tangent." This is the ratio of the length of the leg opposite the angle referred to, divided by the leg adjacent to the angle. Which of these ratios is a tangent? $(\frac{AC}{AB})$ $(\frac{BC}{AB})$ $(\frac{BC}{AC})$

15. Now we have the three important ratios which are the heart of trigonometry: sine, cosine and tangent. There are three more ratios which are just the inverse, or reciprocal, of these ratios. They are called, respectively, secant, cosecant, and cotangent, but we won't try to memorize them. You should memorize sine, cosine and tangent, however, so we'll take 30 seconds to drill.

16. Sine, cosine and tangent are "functions" of a selected angle. They may be considered for our drill as sides of a standard reference triangle with the selected angle at the left, on the base, and the right angle at the right. Which of these is the ratio of the opposite side over the hypotenuse? (sin) (cos) (tan)

17. Yes. Which is the opposite leg divided by the adjacent leg? (sin) (cos) (tan)

18. Right. Which is the opposite side over the hypotenuse? (sin) (cos) (tan)

19. Which is the adjacent side over the hypotenuse? (sin) (cos) (tan)

20. Correct. What ratio of side lengths is the tangent of an angle? $(\frac{\text{opposite}}{\text{adjacent}})$ $(\frac{\text{adjacent}}{\text{hypotenuse}})$ $(\frac{\text{opposite}}{\text{hypotenuse}})$

21. Yes. Now, what is a cosine? $(\frac{\text{opposite}}{\text{adjacent}})$ $(\frac{\text{adjacent}}{\text{hypotenuse}})$ $(\frac{\text{opposite}}{\text{hypotenuse}})$

22. Yes, and what ratio of side lengths is the sine of an angle? $(\frac{\text{opposite}}{\text{adjacent}})$ $(\frac{\text{adjacent}}{\text{hypotenuse}})$ $(\frac{\text{opposite}}{\text{hypotenuse}})$

23. Right. You could repeat this sequence a few times, if you need to practice some more. We have been looking at an angle which increases counterclockwise from the horizontal to the right. On this protractor, you will notice the outside scale increasing in this way. The inside scale, however, starts on the left and increases clockwise. The angle and the ratios remain the same in either direction. To measure an angle, place the angle's vertex at the protractor's center mark, one leg along the lower edge, and read the angle mark where the other leg crosses the scale.

24. Here is part of a table of trigonometric values. Notice that when the angle is zero, the length of the opposite side is zero; and the two ratios, sine and tangent, which use the opposite side in the numerator, will of course become zero. Also, when the angle is zero, the adjacent side which is the numerator for the cosine, will be the same length as the hypotenuse, so the cosine value is one. What is the value of the sine of 30 degrees? (.300) (.360) (.500 (½))

25. Yes, the sine of a 30-degree angle is ½, as you can see in this table. When you study trigonometry at length, it will help to memorize several ratio values, including 0, 30, 45, 60, and 90 degrees. The sine goes from zero to ½, to .7071 to .8660 to 1 for these five values. It is interesting to note that the values for the sine of 45^o and 60^o (.7071 and .8860 respectively) are the square roots of ½ and ¾. This is the result of the relationship noted in the Pythagorean Theorem. What does it say about sides a, b, and c? (they are all equal) (they total 180^o) ($c^2 = a^2 + b^2$)

26. There are a large number of uses for these trigonometric functions, or ratios. Many physical relationships can be surveyed, or measured indirectly, by using them. In recording horizontal angles, for surveys and the like, it is often standard to measure clockwise from North or other reference direction. In measuring vertical angles, the horizontal at that point on the earth is usually used.

27. Vertical angles are often expressed as the "angle of elevation." You could sight along a protractor, or use a transit, or a sextant, to measure the angle from the horizontal to the sun or a star, or the top of a building or pole. If you were on an elevated spot, and depressed your sighting instrument below the horizontal to measure something below your level, what kind of angle do you suppose we would call it? (angle of superposition) (angle of depression)

28. In Ma 10 we solved some problems by suing the length of shadows to determine the height of buildings by setting up similar triangles. We measured the shadow length of some vertical object whose height we knew, to get a ratio, at the time of vertical height to horizontal shadow length. If you measured the elevation angle, what function of it would involve these two legs of your triangle? (sine)(cosine)(tangent)

29. Right. At a given time the sun might be 45 degrees from the horizontal, or at an elevation angle of 45 degrees. This would give you a shadow the same length as the height of the object. What is the tangent of 45 degrees? (.500) (.707) (1.000)

30. Right. But on a cloudy day, or if you couldn't wait until the shadow reached some handy ratio of a reference height, you might use a protractor or transit to measure the elevation angle, then multiply the measured horizontal distance to the object by the tangent of the angle to get its height. The tangent of 13 degrees is .225. What is the height of a building which is 1000 feet away when observed to have an elevation angle of 13 degrees? (225 ft. high) (1000 ft high) (1300 ft high)

31. Yes, and if you know the building was 225 feet high, you could find the point 1000 feet away by moving out until the elevation angle is 13 degrees. Remember that the span between your thumb and middle finger, held at arm's length in front of you, is 14 or 15 degrees, with a tangent of about .250 or ¼. If you walked up the mall toward the U.S. Capitol until the distance between the Washington Monument base door and the top window, about 500 feet high, fills your span, how far away would you be? (500 ft.) (1000 ft.) (2000 ft.)

32. Right. This four-to-one tangent ratio can be quite useful. Another handy tangent ratio is based on the width of your thumb, which held in front of you is about 2 degrees wide, or covers a height or width about one thirtieth of the distance to an object. If you approached a city and noticed that your thumb just covered the height of a 25-story building which was about 250 feet high, how far would you be from it? (750 feet) (250 x 30 = 7500 ft. or 1.4 miles) (7.5 miles)

33. Yes. Sometimes you may know, or need to find, a diagonal distance which would make up the hypotenuse of a right triangle. In this case, which ratio would you use? (tangent) (cotangent) (sine, or cosine)

34. Right. If you were driving across western Kansas on Interstate 70, and noted that the Interstate was heading 30 degrees north of west, how far west, then how far north again would you have to travel to visit Oakley if the bypass rejoined the Interstate five miles further ahead? (4.33 mi. west, 2.5 mi. north) (8.66 mi. west, 7.07 mi. north)

35. Yes. If there is a crane with a boom 50 feet long, and a line extended 40 feet down, what ratio would you first think about to solve some problem using this information? (sine) (cosine) (tangent)

36. Yes. The sine, of course, is the ratio of the opposite side length to the hypotenuse of a right triangle. Sometimes, of course, you don't have a right triangle, but have to create one. This is easy when you have a symmetrical figure, such as an isosceles triangle.

37. The base of this A-frame cabin is easily measured. It is 20 feet wide, and the roof sides slope 60 degrees from the floor. How long are the rafters? (17.33 feet) (20 feet) (26.66 feet)

38. Yes. You mentally dropped a vertical line from the roof ridge, which formed two right angles with base legs ten feet long. The cosine of 60 degrees is ½. Divide the base, ten feet, by the cosine and you get 20 feet for the hypotenuse.

39. You may have guessed whether to divide or multiply by the cosine value, but if you used the step-by-step procedure of algebra, and set up equalities, specified your unknown, solved for it and evaluated it, you would logically find the right answer. What do you think mathematics, and especially algebra, is all about? (a logical system of dealing with quantities) (a random method of confusing the student)

40. Yes. But it's not always easy to learn without some confusion. If you repeat these programs several times, you may learn and remember much more from them. Good studying!

Dorsett Educational Systems, Inc. Box 1226 Norman, Oklahoma 73070 (405) 288-2300